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MATHEMATICAL GEOGRAPHY.

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A COMPENDIUM

OF

MATHEMATICAL GEOGRAPHY.

FOR THE USE OF STUDENTS
AND CANDIDATES FOR PUBLIC EXAMINATIONS.

BY

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WITH A PREFACE BY J. S. LAURIE.

LONDON:
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PREFACE.

If any apology is required for introducing the present work to public notice, a sufficient one appears to be furnished by the fact that no complete special compendium of Mathematical Geography exists in the English tongue. Most of the writers who handle this important branch of science restrict themselves to extremely narrow limits — dealing, for the most part, with this or that particular department, such as map-projections, &c., but not professing to enter into the subject as a whole; while the general treatises to be found in the larger Geographies and Encyclopædias cannot be procured separately, and are therefore practically inaccessible to young students. Again, it is true that in Herschel's 'Astronomy,' for example, the great principles of Mathematical Geography are expounded in a full and masterly manner; but in that and similar works the essential points of the subject are necessarily so interwoven with the details of astronomical science, that they cannot be selected for special study by any but the advanced scholar, and even by him not without considerable loss of time and risk of confusion. The writer of these prefatory remarks having, in common with other students, experienced the deficiency, sought, while pursuing his studies in Germany, for the means of supplying it. He had the good fortune to meet with a clear and concise exposition of the subject in question, viz. Brettner's 'Mathematische Geographie.' The subsequent necessities of his official avocation, however, having rendered a systematic pursuit of the study inconvenient, he transferred the work of translation to a more competent hand. Mr. Dick, who now undertook it, had, as Lecturer on Geography, in a Normal College, the opportunity of testing the value of Brettner's work by actual experiment. He found that the omission by that author of some important branches of the subject, and want of fulness of illustration in others, were such as to preclude the advisability of *translating* the work. He took it, however, as a basis, and, drawing assistance from the best sources, in English, French, and German, he constructed the present volume; which, whether it be viewed as a text-book or as a work of reference, will, it is hoped, fill an acknowledged gap in the library of the student of physical science.

It is believed that the work, as it now stands, will be found to contain all the leading doctrines of the subject of which it treats, arranged in exact and logical order, and expounded with clearness and simplicity. At the same time, the author does not profess to have 'popularised' his subject. For Mathematical Geography can only be 'popularised' (according to the current notion expressed by this term) by presenting its salient points in a form which, while it saves the reader the trouble of mastering the methods of reasoning by which they are established, will yet tend to inspire him with the delusive fancy that he knows the facts as well, or nearly as well, as if he had really undergone that trouble. Such a method of procedure — unfortunately, not seldom adopted — seems to be admirably calculated both to enfeeble the intellectual powers, and also ultimately to destroy in earnest minds every particle of genuine interest in the subject.

J. S. L.

LIST OF THE CHIEF WORKS CONSULTED BY THE AUTHOR.

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The illustrating the obscurities of philosophy with sensible and plausible elocution is not hastily to be condemned, and the thing itself is of great use; for, although it be some hindrance to the severe inquiry after truth and the farther progress in philosophy, that it should too early prove satisfactory to the mind and quench the desire of farther search before a just period is made; yet, when we have occasion for learning and knowledge in civil life, as for conference, counsel, persuasion, discourse, or the like, we find it ready prepared to our hands in the authors who have wrote in this way. But the excess herein is so justly contemptible that, as Hercules, when he saw the statue of Adonis, who was the delight of Venus, in the Temple, said with indignation, 'There is no divinity in thee;' so all the followers of Hercules in learning, that is, the more severe and laborious inquirers after truth, will despise these delicacies and affectations as trivial and effeminate.

BACON: 'Advancement of Learning.'

A COMPENDIUM

OF

MATHEMATICAL GEOGRAPHY.

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CHAPTER I.

PRELIMINARY.

Definitions.

(1.) GEOGRAPHY is a description of the earth. We may consider the earth in relation to the starry sphere which surrounds it; in relation to the structures and operations of nature on its surface; or in relation to its chief inhabitant, Man. Accordingly, geography is usually divided into three branches: mathematical, physical, and political. Mathematical geography treats of the shape, size, position, and motions of the earth in space, and of the different phenomena resulting from these motions. Physical geography regards as its objects all the phenomena connected with the land, the sea, and the atmosphere. While political geography enquires into the divisions of the earth's surface among men, and the constitutions, customs, and commercial relations of the states which these divisions form.

Important lines and points in the heavens.

(2.) Mathematical geography may also be considered as a branch of astronomy; for the earth, as a planet suspended in the starry sphere, is one of the heavenly bodies which are the proper objects of that science. Some of the more important lines and points in the heavens, and the more evident distinctions of the different classes of heavenly bodies, will therefore first demand our attention.

The horizon.

(3.) The circle which, wherever we are, we see surrounding us, forming, as it were, the boundary between earth and heaven, and in whose centre we seem to be, is called the horizon. It is specially named the natural or visible horizon, to distinguish it from the true or rational horizon, which we shall describe by and by. The horizon divides the sphere of the heavens into the visible and invisible celestial hemispheres. The point of the heavens immediately above our heads is called the zenith; that directly opposite to the zenith, in the invisible hemisphere, is the nadir. A circle through the zenith, perpendicular upon the horizon, is called a vertical circle.

Zenith.

Nadir.

Apparent movements of the sun.

(4.) The sun appears in the morning above the surface of the earth, and gradually rises higher above the horizon till it attains its greatest height at mid-day. It then begins to sink towards the opposite side of the horizon, and in the evening vanishes below it, at a point exactly opposite to that at which it rose. On the following day it again appears in the part of the heavens where it appeared the day before, and thus seems to have described a circle in the heavens, one half of it above the earth in the day time, the other half below, at night. This circle it repeats daily, always taking about twenty-four hours to perform it, and always moving in the same direction. The part of the heavens in which it rises is called the east, that where it sets, the west. The part in which we see it at its highest point above the horizon is the south; the opposite part, a part in which inhabitants of Europe never see it, is the north.

East.

West.

South.

North.

(5.) The sun does not always rise exactly above the same point of the horizon, neither is it exactly at the same height above it every day at mid-day. That point of the eastern heaven at which it rises on March 21, and September 23, is called the east point; and the west point is that point of the western heaven at which it sets on those days. If we join these two points by an imaginary line, and then imagine another line drawn perpendicular to the first through our position and extended on both sides to the horizon, the points of the horizon touched by the second imaginary line are the north and south points.

The meridian.

(6.) A circle in the heavens which passes through the zenith and the north and south points of the horizon is called a meridian circle, from *meridies* — mid-day — because when the sun, in its apparent course from east to west, goes through this circle, it is mid-day.

Apparent movements of the moon.

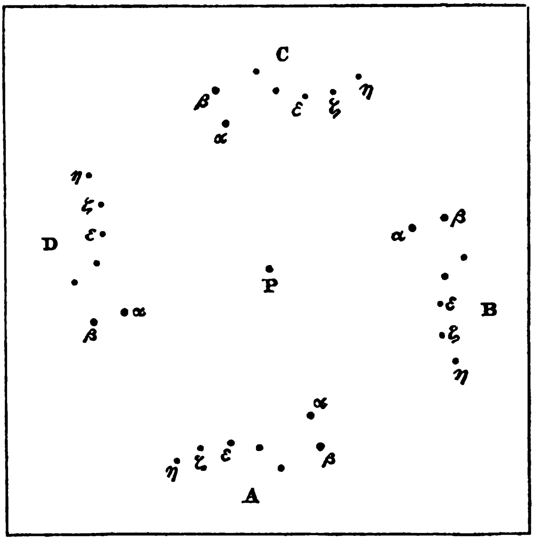
(7.) The moon has a motion similar to that of the sun. It rises in the east, and passes over the south towards the west, where it disappears below the horizon.

Apparent movements of the fixed stars.

(8.) A little observation suffices to show us that the stars preserve their relative positions towards each other unchanged; so that if the imaginary lines joining any three stars form a right-angled triangle, they always preserve that form. But although the stars do not change their relative position, every star moves slowly and steadily in the same general direction as the sun and moon, from east to west; and, indeed, when closely watched, they are seen to move in concentric circles, these circles having for their common centre a point in the northern heavens near to the star called the North Polar Star. This star is easily found by means of that well-known constellation the Great Bear. The seven brightest stars in that constellation (whose relative positions are represented in fig. 1) form the tail and hind quarter of the imaginary animal. They are often called Charles's Wain, sometimes the Plough. The last name seems the most appropriate to the figure they form. The two stars marked α and β are called the pointers, because they always point to the Pole Star. An imaginary line joining them, and prolonged about five times the distance between α and β, will reach the Pole Star; which is very near the actual pole or turning point of the heavens. Being so near, it describes

Of the Great Bear round the pole.

Fig. 1.



only a very small circle round the pole—so small that its motion is not easily observable, and it is, therefore, often looked upon as the very North Pole. A single evening's watching of the stars in the Great Bear will strikingly show the motion of the heavenly bodies. At first, for instance, about five or six o'clock of a winter's evening, the seven principal stars of the constellation will be seen beneath the North Polar Star, as at A, with the three stars forming the tail horizontal and pointing towards the west; about six hours after they will be seen east of the Pole Star, with the tail turned downwards, as at B. Twelve hours after they will be directly above the Pole Star, with the tail horizontal once more, but pointing towards the east, as at C; and eighteen hours after, if they could be seen (as they can through a telescope), they will be west of the Pole Star, with the tail turned upwards, as at D. Twenty-four hours after the first observation, they will be seen again in the original position. It is as if the stars were fixed in the vault of heaven, and this vault turned round daily on an axis, the position of which is an imaginary straight line joining the North Pole with the diametrically opposite point of the celestial sphere—that is, with the part of the heavens called the South Pole. That great circle of the heavens upon the plane of which this imaginary axis is perpendicular, and from which, therefore, both the north and south polar points are equally distant (namely, 90°), is called the Celestial Equator or Equinox.

The axis of the celestial sphere.

The celestial equator.

Annual movement of the sun and moon among the stars.

(9.) All the stars, without exception, move from east to west about the axis of the celestial sphere in twenty-four hours. On closer observation, however, of the sun, the moon, and *some* stars, we perceive that these have also a second motion in the contrary direction, from west to east. In the moon this motion is greatest. This is easily perceived if we watch that luminary as it sets, for several successive days, and observe the stars that are near it at the time of setting. We then perceive that different stars set with it every day—that, in fact, those which were six-and-twenty full moons' breadths eastward from it at the last time of setting, set with it the next; and, that, next again it sets with those which had been the same distance still farther east. After rather longer than twenty-seven days, it again sets with the same stars as at first, and thus appears to have made a circuit backward in the heavens from west to east.

The moon's progress among them.

The sun's.

(10.) The sun moves daily in the heavens with the same backward motion among the stars, but at a much slower rate than the moon, namely, about two suns' breadths daily, so that to come round to set again with the same stars, it requires something more than 365 days. Its daily progress among the stars we know from this: that stars which to-day, as it were, are in the south part of the heavens when the sun is setting in the west, and must, therefore, set in the west six hours after it, to-morrow set less than six hours after the sun, the next day less still, till, when a quarter of a year has elapsed from the first observation, these stars set with the sun. Further, it is observed that stars which to-day appear in the east soon after sunset, and about six hours after are in the south, on the lapse of a quarter of a year are in the south at the time of sunset, and that, when a second quarter has gone, they set with the sun. Thus, the stars which appear in the west shortly after sunset are constantly changing — they disappear with the sun, while new ones are constantly rising in the east — so that, after about 182 days from the first observation, the starry heavens appear completely changed; and, again, after 365 days, the state of the stars in relation to the sun is the same as before. The circle in which the sun thus appears to move backward among the stars in a year, from west to east, is called the Ecliptic, or Sun's Path.

Of the two motions thus observed both in the sun and moon, the daily motion from east to west is called a *retrograde*, that from west to east, among the stars, a *direct* motion.

(11.) Besides the sun and the moon, there are some other stars which change their relative place in the heavens, with no apparent law to regulate their movements. The ancients, from this circumstance, called them by a name which means wandering stars, planets, and reckoned the sun and moon in the same class. The stars which never change their relative position, and are, as it were, *fixed* to the vault of heaven, they called fixed stars. By planets we now mean bright heavenly bodies, which move in ellipses about the sun, receiving from it their light and heat. Smaller, less easily perceived heavenly bodies, which move about the planets in ellipses, and accompany them in their paths round the sun, we call satellites, or moons. Besides planets and fixed stars, there are also comets, which, indeed, like planets, move around the sun, but are clearly distinguishable from planets, partly by their appearance, as they have commonly a long shining tail on the side turned away from the sun; and partly by their path, as they move in greatly elongated ellipses, and in all quarters of the heavenly sphere.

Planets.

Fixed stars.

Satellites.

Comets.

Difference between planets and fixed stars.

(12.) There are other differences between planets and fixed stars. The fixed stars give a twinkling and restless, the planets a clear and steady light. The light of the fixed stars is like that of the sun, of the planets like that of the moon. The planets appear larger through a telescope than to the naked eye; the fixed stars are unchanged in size. The fixed stars are scattered through all parts of the heavens; the planets move always in a small belt of the sphere which extends about ten degrees on each side of the ecliptic, and is called the Zodiac.

The solar system.

(13.) The sun is one of the fixed stars; the earth is a planet; the moon a satellite. Sixty-seven planets, twenty-one moons, and very many comets are now known. There are three groups of planets classed according to their distance from the sun; viz., the inner, the middle, and the outer group. Of the inner planets, which are all of a medium size, there are four, namely, Mercury, Venus, the Earth, and Mars. Of the middle group, distinguished by their comparatively small size, there are at present known fifty-six, and new discoveries are constantly adding to their number. The names of the four longest and best known are Ceres, Pallas, Juno, and Vesta. To the outer group, that of the largest planets, belong Jupiter, Saturn, Uranus, and Neptune. The Earth has one moon, Jupiter four, Saturn eight, Uranus six, and Neptune two.

Its groups of planets.

(14.) The sun, in the centre, with the comets, planets, and their moons ever revolving round it, form what is called the Solar System. The fixed stars, divided according to their apparent brightness, into stars of the first, second, third, &c., to the sixteenth magnitude, hang above us in countless numbers. When less than of the sixth magnitude, they are invisible to the naked eye. Of the first seven magnitudes Herschel estimates the number to be about 20,000. Every increase of the power of optical instruments has revealed myriads before invisible. Each of these fixed stars is a sun — forming probably the centre of a distinct solar system — with planets, satellites, and comets revolving round it. The sum of these countless solar systems is the universe.

The number and magnitude of the fixed stars.

CHAPTER II.

THE FORM OF THE EARTH.

The earth round.

(15.) THE surface of the earth is not an extended plane as it seems to be. The earth is a globe or sphere, and therefore its surface is curved. The proofs that the earth is a globe are derived partly from observations of phenomena upon the earth itself—partly from similar observations in the sphere of the heavens. The first may be called terrestrial proofs of the earth's rotundity, the others celestial. Of the ten proofs which follow, four are terrestrial; six celestial.

Terrestrial proofs.

First: from the appearance of distant objects.

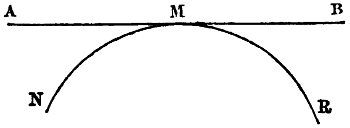
(16.) The first proof which we shall adduce is derived from the appearance presented by elevated objects as they are approached or departed from. If we are in the midst of a plain, and direct our attention to a steeple, a tower, or any tall object in the distance, we see only the upper part of it, and more or less of that according to our distance from it. If we approach it, the part seen becomes greater, and the increase takes place *from below*. If we depart farther from it, the part seen becomes less, and the decrease takes place from below—that is, the lower parts become hidden. This increase and decrease from below is a consequence of the earth's surface being curved like that of a globe. As we recede from the object, the earth rises up, as it were, between us and it, and finally hides it altogether. As we approach it, we surmount the earth which has interposed between us and it; till at last we stand upon a level with it, and see the whole. Were the earth a plane, then so long as the visual power of the eye was strong enough to see the object, or the density of the intervening atmosphere did not hide it, we should see the *whole* of any such object. The only difference which would occur through variation of distance, would be, that the object would appear to increase, as a whole, if we approached it, and to decrease, as a whole, if we receded from it.

(17.) The case is similar at sea. A ship departing from the land is seen, by the observer from the shore, to sink, as it were, beneath the level of the sea. It goes down behind the offing, or horizon of the sea. The part last seen is the top of the mast.* Had the vessel been receding on a plane, it is evident that the part longest seen should have been that of largest bulk, namely the hull. To prove that, though the hull has disappeared, it is not because it is beyond the range of vision, let the spectator, immediately after losing sight of it, quickly ascend a height, so as to extend his horizontal view, the hull will again come into sight. As he descends he will again lose it, while yet the pennon at the top of the mast may be clearly visible above the interposed segment of the sea. In like manner, when two vessels at sea approach each other, the sailors in each at first see only the upper rigging of the other, gradually the masts rise as they approach, and at last the hulls are visible above the offing. Now these appearances are the same in whatever direction a tower or a vessel is approached or departed from: the earth therefore must be curved in all directions.

Second proof: from the circular form of the horizon.

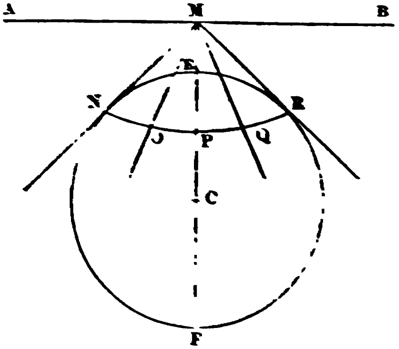
(18.) The second proof of the earth's globular form is derived from the constant circular form of the horizon, and its constant increase in a circular form as the point of observation becomes higher. Hills, rocks, and trees prevent our forming a clear idea of the true shape of the horizon on land: only on some high eminence in the middle of a vast plain, or on a small island where the sea is visible on all sides, can this be attained. At sea, the horizon is always a distinct clearly defined circle, which becomes the more extensive the higher our point of vision is: for instance, its extent is greater when it is viewed from the main truck than from the deck, still greater when seen from the topmast, greatest of all from the top of a very high mountain, like the Peak of Teneriffe. The portion of the earth's surface visible to us at any time is that which lies above a tangent drawn from the eye to the earth. At the level of the sea, therefore, it is only the parts immediately surrounding us that we can see, namely, those along and above a line at right angles to our position. In fig. 2, if NMR represent a portion of the earth's surface, and AMB be a tangent to it; to a spectator at M only the parts of the earth above AMB will be visible. As we rise higher by climbing the mast of a vessel, or to the summit of a mountain, the tangent from the eye to the earth MN (fig. 3), inclines downward, forming the angle AMN with the line AB at right angles to our position. This angle AMN is called the dip of the horizon. Suppose the line MN to be carried round from the spectator's position as a centre into the successive positions MO, MP, MQ, MR, then the point of contact N will describe on the earth's surface the circle NOPQR, which will be the boundary of his view on all sides. The area of this circle, therefore, is the portion of the earth's surface visible from M. It is evident that the higher the point M is, the greater will be the dip of

Fig. 2.



the horizon, and the greater the portion of the earth included within the circumference of NOPQR. In all parts

Fig. 3.



of the earth the horizon forms such a circle. Now the figure of a body, the boundary of which is always circular, can be no other than that of a sphere or globe. The greatest extent of the earth's surface ever seen by man at one time, was that exposed to the aëronauts Coxwell and Glaisher, who in September 1862 ascended to a height of more than six miles in a balloon.

(19.) We may calculate the area of the circle spread out before these aëronauts by taking MN as the radius of their circle of view NOPQR, and the height of their position ME (fig. 3) $= 6\frac{1}{8}$ miles.

By Euclid III. 36, $MN^2=(EF+ME)\times ME$, EF being the diameter of the earth. As ME is very small compared with EF we may neglect it, and we have $MN^2 = EF \times ME$. Assuming, for ease of calculation, that the earth's diameter EF is 8,000 miles, we have $MN^2 = 8{,}000 \times 6\frac{1}{8} = 49{,}000$; and therefore $MN = \sqrt{49{,}000} = 222$ miles nearly, which is the length of the radius of the circle NOPQR.

Now, area of NOPQR $= (\text{diameter})^2 \times \cdot 7854 = (444)^2 \times \cdot 7854 = 154{,}830$ square miles.

Again, the area of the earth's surface $=(8{,}000)^2 \times 3{\cdot}1416$ $=201{,}062{,}400$ square miles. Hence we have area of earth : area of circle of view $= 201{,}062{,}400 : 154{,}830 = 1{,}300 : 1$, very nearly; that is, from the height of somewhat above six miles these daring explorers of the air were enabled to overlook $\frac{1}{1300}$th part of the earth's surface.

Similarly, it may be found that the area visible from the Peak of Teneriffe, 12,000 feet high, is $\frac{1}{4000}$th part of the whole surface of the earth.

Third proof: from the early disappearance of objects in the distance.

(20.) A third proof, which shows at least that the earth is not a plane, may be found in the earlier disappearance of objects at a distance than should happen if the earth were a mere plane.

The limit of visibility of a white object upon a black ground is generally considered to be attained when the object subtends an angle of forty seconds at the eye. Let us take the case of a mountain relieved against the sky, and allow a three times greater angle as the limit of distinct vision, namely, an angle of two minutes. Even making this allowance as a set-off against the blending colours of the sky and mountain, a mountain 5,000 feet in height should still be visible at the distance of 1,627 miles. But experience shows that it is not so, plainly because it is hidden by the curvature of the earth.

(21.) Let h be the height of the mountain, d the greatest distance at which it can be seen on the earth, a the angle of vision, then is $\frac{h}{d}=\tan a$.

Thus, for $a=2$ minutes, and $h=5000$ feet, we have $\frac{5000}{d}=\tan 2' \therefore d=\frac{5000}{\tan 2'}$ and $\log d=6{\cdot}934214$.

Hence $d=8{,}594{,}364$ feet, or about 1,627 miles.

Fourth proof: from circumnavigation.

(22.) The fourth proof of the earth's rotundity is found in the actual circumnavigation of the globe.

It is a well-known fact that navigators, preserving unchanged the general direction of the course in which they set out, have again returned to their original starting point. Sailing constantly to the west, they have found their way home again arriving from the east; and, on the contrary, sailing constantly in an easterly direction they have arrived from the west. On a plane, return would be impossible, except by turning the ship's prow in a contrary direction from that which it had at starting. Or if, indeed, a circular voyage were made, it could only be by continually changing the direction of the vessel. Magellan first made a voyage round the world, during the years 1519-22. Drake did it again sixty years later. Cook and others, in approaching as nearly as they could to the polar circle, have found that the distance round the world was always less the nearer they were to the poles; and have thus afforded a probability that the earth is round from north to south, as well as from east to west. The ice that environs the poles prevents an experimental demonstration of its rotundity from north to south.

Celestial proofs.

Fifth proof: from the difference in time of rising and setting of sun, moon, and stars at different places.

(23.) We now turn to the consideration of some of the phenomena of the heavenly bodies, which serve as proofs that the earth is a sphere. Of these, the first we notice, is the fact that at places eastward of us a star or the sun rises and sets earlier than it does at our station, and at places west of us it rises and sets later. Were the earth a plane, then the inhabitants of all places should see a star or sun set at the same time. To all places therefore, day and night should commence at the same time. Experience shows that this is not the case. Places east of us have the commencement of day and night earlier than we have, and places west of us later. This can only be accounted for by the curvature of the earth. The farther we are to the west, from the sun which is approaching us from the east, so much the greater is the part of the earth's curved surface interposed between us and the sun, and hiding it; and so much the longer will he be in coming round above our horizon. The more eastward we are the sooner we have him in any given day.

Three consequences of this difference.

(24.) Three consequences of this should be noted:—

Time differs at different places,

1. If we carry a watch set correctly to the time in our own locality to any place westward of ours, it will show time before that of the new place; since it shows the time of a place which necessarily has the hours of sunrise, midday and sunset before these can arrive at the locality farther west. The watch will, therefore, appear to have gained time. If it be carried to a place farther east than that to the time of which it is set, it will for the opposite reason be behind the clocks there, and appear to have lost time.

Days are shortened or lengthened by travelling east or west,

2. The traveller from east to west makes his day longer than if he stood still; the traveller from west to east makes it shorter. In the first case he goes in the same direction as the sun; it is, therefore, later in passing over his meridian and setting in the western horizon, and the day must have closed to those at the locality he has left before it closes to him. When he travels east he goes towards the sun; it passes over his meridian sooner than if he had remained still, sets to him sooner, and thus shortens his day.

One day's reckoning is lost or gained by sailing round the world.

3. Those who sail round the world lose a day in their reckoning of time if they sail westward, and gain one, if eastward. To explain this we must anticipate the proofs, afterwards detailed, that the earth revolves. It is this revolution from west to east which causes the sun to appear as if rising in the east and setting in the west, bringing the changes of day and night. So

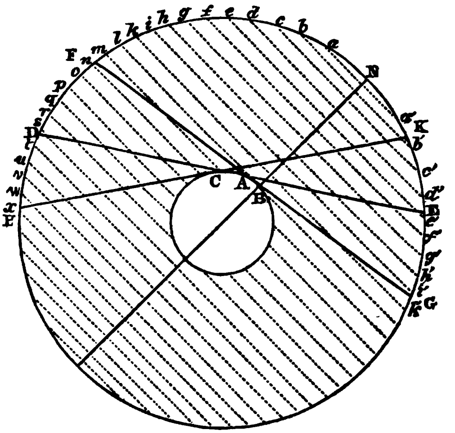
many turns round its axis as the earth makes in a given time, so many times does the sun pass over and descend below our heaven, and so many days are reckoned. A voyager sailing round the globe in the same direction as it moves, no matter how long the voyage, must, in addition to the number of times in which he has turned with the globe and experienced successions of day and night, have made one turn more by the motion of the vessel, and therefore must have witnessed one day and night more than if the rotation of the earth alone had carried him round. He will, therefore, count one day in advance of his friends at home — his Sunday will be their Saturday. If he sail against the direction of the earth's motion, that is westward, he will, in effect, be resisting the earth's power of carrying him round, and, when he has circumnavigated the globe, will have made one revolution less than the earth. On his return, therefore, his Sunday will be the Monday of his fellow-countrymen. This confusion of days has actually occurred to circumnavigators.

Sixth proof: from the changed position of the stars in different latitudes.

(25.) The changes which are observable in the numbers and orbits of the stars as we travel from north to south, or from south to north, prove that the earth is round.

We have already described the daily revolution of the stars round the north pole (8). As this point of the heavens in our latitudes is not in the zenith nor yet on the horizon, but between the two, it is evident that the circle they describe round it must be inclined towards the horizon. Some near it, like those of the Great Bear, describe the whole of their circle about the pole above our horizon and never set. Those a little farther off describe about half of their circle round it above our horizon; others are seen by us only during a small portion of their orbit. If we travel northward, that is, in a direction towards the polar star, it rises higher above our horizon, and some of the stars, which were wont to set at our former position, are now seen during the whole of their daily revolution, while others have disappeared altogether from the southern heavens. The pole star being now higher up, the orbits of all the stars are now less inclined to the horizon than before. But if we travel south, the pole star sinks towards the horizon; fewer stars than at first are visible during the whole of their revolution, and new stars are continually appearing above the southern horizon. The orbits, too, of all the stars are now more inclined to the horizon than at our first station. If the earth were a plane, it is evident that in every situation upon it we should always see exactly the same stars, and that in all situations they would preserve the same inclination to the horizon. On the other hand, let us suppose that the earth is a sphere, and in fig. 4 let the smaller circle

Fig. 4.



represent the earth while the larger represents the sphere of the heavens. Let N represent the north pole, A our position on the earth, and DE the horizon at our position. It is evident from the figure that all the stars above DE will be visible to us at A. NE will be the height of the north polar star above the horizon, and all stars nearer the pole than E will have the whole of their daily circle above DE. The orbits of these stars are represented by the dotted lines *aa*, *bb*, *cc*, &c, and their inclination to the horizon DE is evident. Let B be a place to which we travel north of A, and FG its horizon. All the stars above FG will be visible at B; and the curve GN, larger than NE, will be the height of the pole star in the new locality. All the stars nearer the pole than G will be seen from B in the whole of their daily circle, while those formerly seen at A, between D and F, will be altogether invisible. Again, let C be a place to which we travel, lying southwards from A, HK its horizon, and the height of the pole star the diminished curve KN. Only the stars nearer the pole than K will be visible in the whole of their daily orbit, but new stars between D and H will have risen to view. Thus, on the supposition of the spherical form of the earth, the changes which occur are easily explained.

The eclipses of the moon afford another proof.

Seventh proof: from eclipses of the moon.

(26.) An eclipse of the moon takes place when the earth, in its progress round the sun, comes so between it and the moon, that the latter in its course must pass through the cone of the earth's shadow. The position of the earth, in reference to the sun and moon, is very different during different eclipses, yet, however it may differ in position, the edge of the earth's shadow as it falls on the bright face of the moon is always circular. This proves the earth to be a sphere, for only a sphere throws always a round shadow, whatever be its relative position to the body on which the shadow is cast and to the source of light. A cylinder, a cone, a flat round disc, may all, indeed, be made to cast a round shadow, but only in certain positions.

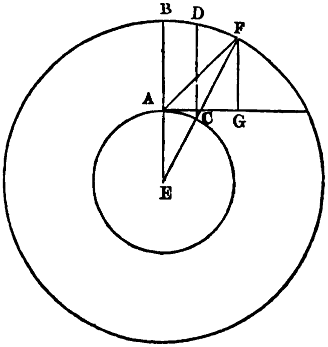
Eighth proof: from analogy of the planetary forms.

(27.) An argument of analogy for the spherical form of the earth is derived from the spherical form of the planets. Astronomers, by means of the telescope, see that the other planets have a spherical form. Certain appearances in these bodies, such as dark spots appearing at one edge, traversing their surface, then disappearing, and after a certain time reappearing to go the same round again, show that the planets have a motion round their own axis. The parts presented to us are thus constantly changing, yet amid all these changes the planets always appear as circular discs. They must, therefore, have a spherical form, for, if not, they would, at times, appear only as a line. These planets, so like the earth in other respects, being spherical, we are naturally led to infer that the earth must be so too.

Ninth proof: from correspondence of arcs in the heavens with distance on the earth.

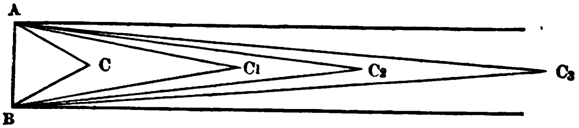
(28.) Another proof is found in the correspondence between arcs in the sphere of the heavens and distances on the surface of the earth when measured along a meridian.

Fig. 5.



In fig. 5, let the smaller circle represent the earth, the larger the sphere of the heavens. Let a star at B be in the zenith of the place A, while at C, a locality on the same meridian line AC, the point F is in the zenith. From C, the star B will be seen along the line CD parallel to AB; for, on account of the immense distance of the fixed stars, lines from any part of the earth, even the most distant, to the same star are parallel. Therefore at C, the star B will make the angle DCF with the zenith; that is, it will be distant from F, the zenith, by the number of degrees in the arc DF. But the angle DCF is equal to the angle AEC, since CD is parallel to EB; and if the earth be a globe, and therefore a meridian line a circle, DF and AC will each contain the same number of degrees of their respective concentric circles. Arcs in the heavens equal to DF will, therefore, be subtended on the earth by distances equal to AC. If then, supposing the earth a globe, the distance between A and C be measured at any two other places on the same meridian, separated by a distance equal to AC, there will be a difference in the zenith distances of the same star equal to DF. This is found to be actually the case all over the world. By travelling in any part of the earth, about 69½ English miles due north or south, a star which when it crossed the meridian was in the zenith of the place we have left, will be found, on crossing it at the new locality, to be one degree from the zenith. The earth, therefore, must be a globe.

Fig. 6.



(29.) Fig. 6 may serve to illustrate the truth that the lines along which a fixed star is visible from different parts of the earth's surface are parallel. Let A and B be two distant points upon the earth; they may be considered as extremities of a diameter of the earth. Let C C_1 C_2 C_3 &c., be an object successively removed to different distances from the line joining A and B. The lines drawn from A and B to C, the nearest distance, converge; those drawn to C_1 more distant, converge less; those to C_2 still less; to C_3 still less, &c., till at last we may conceive C_n to be removed to a distance so great, that the convergence of AC_n and BC_n will be imperceptible; they will, therefore, be practically parallel. Astronomy shows that the fixed stars are actually at such a distance from the earth.

Tenth proof: from the zenith distance of a star being easily made less.

(30.) Our last proof that the earth is a globe, is derived from the short journey required in order that a star near the zenith may be brought on to it.

In fig. 5 let the star B be in the zenith; then, in order that the star F may be brought upon it, supposing the earth a globe, the curve AC only will have to be traversed. But if the earth were a plane we must pass over the straight line AG for this purpose, FG being a perpendicular from the star F upon the extended plane surface of the earth, and AG a line upon that surface from A to the point immediately under the star. Suppose BF to be an arc of 1° in the celestial sphere, and the radius of the earth to be 4,000 miles, then AC is found by experience to be 69½ miles. On the contrary, supposing the distance of the nearest fixed star to be a billion of miles (and we shall show in a future paragraph that it cannot be less), then the line AG to be traversed, if the earth were a plane, would be above 17,000,000,000 of miles.

For AG=AF cos FAG=1,000,000,000,000 times cos. 89° =1,000,000,000,000 times ·0174524=17,452,400,000 miles.

This result all experience contradicts.

Ancient ideas of

(31.) The ideas of the ancients on the figure of the

the earth's figure.

earth were absurd enough. To Homer it appeared a circular level washed on all sides by the many-sounding waves; to Thales of Miletus it was a flat disc floating on a boundless ocean. Anaximander thought it a cylinder inhabited on its upper end; and Plato reasoned that it must be a cube, that being the most perfect solid, as he thought. Pythagoras seems to have guessed at its true shape; and Aristotle gave the sixth and seventh of the foregoing proofs from observations of his own. But ignorance and theological prejudice prevented the fact of the spherical shape of the earth from being generally believed, till the period of the reawakening of the human intellect about the fourteenth century, when the polarity of the magnet was discovered, printing invented, and the inductive method of reasoning adopted. The belief of this truth led Columbus to the New World; and, by acting on it, the navigators of the sixteenth century were enabled fearlessly to explore unknown regions, instead of creeping timidly along familiar shores as heretofore. The commerce of Europe was thus extended, intercourse between distant nations became frequent, knowledge was increased, and prejudices were broken down. The revival of the long-forgotten belief in the roundness of the earth may therefore be considered as one of the causes of modern civilisation.

CHAPTER III.

POINTS, CIRCLES, AND POSITIONS ON THE CELESTIAL SPHERE.

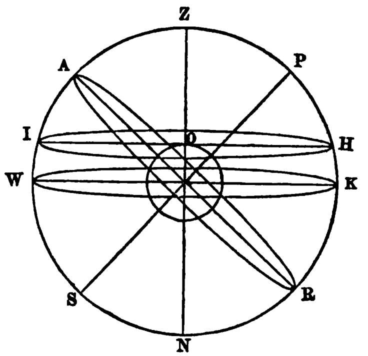
The spheres of the heavens and the earth correspond.

(32.) THE earth is a sphere. When we come to consider its magnitude we shall find that the irregularities upon its surface are proportionally too minute to render necessary any modification of this statement. In fact, they are almost as nothing compared to the whole mass of the earth. As a sphere, the consideration of many particulars concerning the earth belongs to the science of spherical geometry. Astronomers have marked out certain points and circles, by the aid of which we arrive at more definite ideas respecting the earth's motions and position in space. These circles and points are not arbitrarily assumed, but are derived from observations of the phenomena of the heavens. The mathematical divisions of the earth's surface are only reflections as it were of the divisions of the heavenly sphere — features of the concave surface of the celestial, transferred to the convex surface of the terrestrial sphere. We have already said as much concerning these points and circles as was necessary for the understanding of the former sections: our future course will require some farther explanation of them, during which a little repetition may be unavoidable, perhaps useful.

The horizon: visible and rational.

(33.) In fig. 7 the small circle represents the earth, the larger one the celestial sphere, O, a place upon the surface of the earth. The circle IH, drawn in perspective, represents a plane touching the earth at the point O; and extended on all sides to the celestial sphere. WK is an imaginary plane through the centre of the earth parallel to IH, and similarly extended. IH is called the visible or natural horizon (3), and WK the true or rational horizon of the place O. Although these two horizons are as wide apart as the length of the earth's radius, about 4,000 miles, yet they are regarded as one, and practically are so, because the space by which they are separated is as nothing at the distance of the fixed stars. A star, therefore, which is on the visible horizon is at the same time on the rational horizon. The circle, which in an open plain or at sea, we see as the boundary between earth and sky, and in whose centre we seem to stand (3), is now neither IH nor WK, but one, as it were, between the two, and so much the nearer the true horizon WK, as our place of observation is higher. The horizon of every place divides the celestial sphere into the visible hemisphere WZK, and the invisible WNK. The pole of the horizon, or the extreme point of its axis, in the visible hemisphere, or the point exactly above our heads, is the zenith Z; the opposite point of the celestial sphere, that point of it exactly beneath our feet, is the nadir N.

Fig. 7.



The poles

(34.) The consideration of the daily motion of the stars leads to that of the central point of their orbits (8). This point and that opposite to it in the celestial sphere, are called the poles of the heavens, and the imaginary straight line joining these poles, around which the celestial sphere seems to revolve, is called the axis of that sphere. A great circle in this sphere, having the same axis as the sphere, and for its poles the extreme points of that axis, is called the celestial equator, or equinoctial. Its circumference divides the celestial sphere into two equal parts, and is so situated in the sphere that every point of it is at the same distance from the two poles, namely 90 degrees. The pole visible in Europe is the North, or Arctic Pole, that directly opposite is the South, or Antarctic. Of the two hemispheres marked off by the equator, that in which the North Pole is above the horizon is the northern — the other the southern hemisphere.

and the axis of the celestial sphere.

The equinoctial.

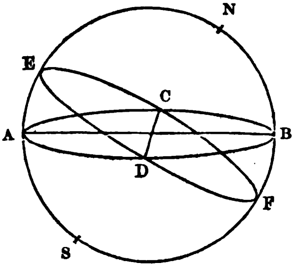
(35.) The great circle of the celestial sphere which passes through the zenith of a place and through the North Pole, is called the celestial meridian, or mid-day circle of that place; and is so named because the sun crosses that circle at mid-day. The same circle passes through the South Pole and the nadir of the place; its plane, therefore, must pass through the axes of the horizon and the equator, and must be consequently perpendicular to the plane of the equator: hence, too, the equator, the horizon, and meridian circle must each bisect the other two. The intersections of the circumferences of these three circles form important points in the study of the earth's surface. Of the two points of intersection of the horizon and meridian of a place, that which is nearer the North Pole is called the north point, and the more remote the south point. The straight line joining these points, being the line of intersection of the planes of the horizon and meridian,

The celestial meridian.

may be called the terrestrial meridian of the place. Of the two points of intersection of the circumferences of the equator and the horizon, the one which lies to the right of an observer looking north is called the east point; that to the left is the west point. These four points divide the circumference of the horizon into four equal parts, so that each point is 90° distant from the points next to it on either side.

In fig. 8 let ACBD be the horizon, ECFD the equator, AENBF the meridian, a line joining N and S the axis of the sphere, N the North Pole, and S the South; AB will be the line of intersection of the planes of the horizon and the meridian; and CD the line of intersection of the planes of the horizon and the equator; then B will be the north, D the east, A the south, and C the west point, and AC = CB = BD = DA = 90°.

Fig. 8.

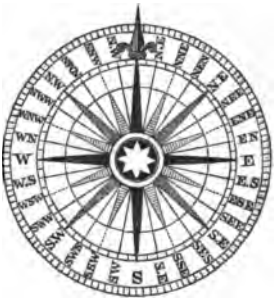


Points of the compass.

(36.) These four points of the horizon—north, south, east, and west — are called the cardinal points of the horizon, more commonly the cardinal points of the compass.

The intermediate points are found by repeated bisections of the arcs AC, CB, BD, DA. The resulting points from the first bisections are called north-east, north-west, south-east, south-west. The bisection of the arcs between the points now found gives the new points north-north-east, east-north-east, north-north-west, west-north-west, south-south-east, east-south-east, south-south-west, west-south-west. Again, bisecting the arcs between the points now found, we have (using only the initial letters of the four cardinal points) N. by E., N.E. by N., N.E. by E., E. by N., E. by S., S.E. by E., S.E. by S., S. by E., S. by W., S.W. by S., S.W. by W., W. by S., W. by N., N.W. by W., N.W. by N., N. by W. The whole number of points thus found is thirty-two. A circular card with these thirty-two points marked, in the centre of which is a pivot bearing a freely turning magnetic needle, forms the mariner's compass.

Fig. 9.



(37.) As the east and west points of the horizon are situated on different sides of the meridian, we may define the meridian as that great circle of the celestial sphere which divides it into the eastern and western hemispheres. These two points of the horizon are, in fact, the poles of the meridian. The passing of a star in its daily motion through the heavens over the meridian is called its culmination. Every star has two culminations daily, an upper and an under — the one above and the other below the horizon — unless the star be one which has the whole or none of its path above the horizon. In the first case, both of its culminating points are above, in the other both below, the horizon. A star is always at its greatest height above the horizon when in its upper culminating point. The sun culminates above the horizon at midday, and under it at midnight.

Upper and lower culminations of a star.

(38.) The path which the sun, in its motion from west to east among the stars, describes in a year is called the ecliptic. The ecliptic and equator intersect at an angle of 23½° (more exactly 23° 27′ 41·4″).

The ecliptic.

This angle is called the inclination of the ecliptic. By the amount of this angle the ecliptic rises above the equator towards the North Pole, and sinks below it towards the South Pole. On account of its oblique position it was called by the Greeks the oblique circle. We are most easily led to the understanding of this obliquity, by observing that the sun is not at the same height above the horizon every day; and to a knowledge of its varying amount, through observing that in spring and autumn (March and September) it is, when it culminates, at the same height as the stars on the equator, that is (in the latitude of London) 38½° above the horizon; but in summer it is 23½° higher, and in winter 23½° lower.

Its inclination changes.

The inclination of the ecliptic in the year 1800 was exactly 23° 27′ 54·8″, and every year this inclination diminishes about half a second (more exactly 0·4758″) owing to the effect of the stronger attraction of the planets, especially Jupiter and Venus, upon the parts of the earth about the equator, than upon the other parts.

The equinoxes.

The solstices.

(39.) Four points on the ecliptic demand our attention; namely, the two points of intersection of its circumference with that of the equator, and the two points of its circumference farthest from the equator. The first two are called the equinoctial points. One of these, through which the sun passes towards the northern part of the celestial hemisphere, is called the vernal or spring equinox; the other through which it descends towards the South Pole is the autumnal equinox. The two points on the ecliptic farthest from the equator are called the solstices. That nearest the North Pole is the northern or summer solstice; the other the southern or winter solstice. The reason of these names will be seen hereafter. The great circle of the sphere which passes through both equinoctial points and the poles of the heavens is called the equinoctial colure; and that which passes through the solstices and the poles is called the tropical or solstitial colure.

The colures.

It will be easily seen that: —

1. The arc of the solstitial colure between the ecliptic and the equator is the measure of the obliquity of the ecliptic.

2. The poles of the equator and those of the ecliptic are separated from each other by an arc of a great circle equal to the obliquity of the ecliptic (23½°).

The signs of the zodiac.

(40.) These four points of the ecliptic divide its circumference into equal arcs, each therefore 90°. If we subdivide each of these arcs into three equal parts of 30°, we obtain the so-called twelve signs of the zodiac, named after groups of stars which *formerly* (about 2,200 years ago) stood in these divisions. The signs, reckoned from the vernal equinox, follow each other thus: —

SPRING	♈ Aries (the Ram), March 21.
	♉ Taurus (the Bull), April 19.
	♊ Gemini (the Twins), May 20.
SUMMER ...	♋ Cancer (the Crab), June 21.
	♌ Leo (the Lion), July 22.
	♍ Virgo (the Virgin), August 22.
AUTUMN ..	♎ Libra (the Balance), September 23.
	♏ Scorpio (the Scorpion), October 23.
	♐ Sagittarius (the Archer), November 22.
WINTER ...	♑ Capricornus (the Goat), December 21.
	♒ Aquarius (the Waterman), January 20.
	♓ Pisces (the Fishes), February 19.

The first six, being north of the equinoctial, are sometimes called the northern signs, the other the southern.

Different now from the constellation which gives the name to them.

(41.) At the period in which these names were bestowed, about 2,200 years ago, the group of stars, or *constellation* of the Ram, did actually stand in the *sign* of Aries; that is, in the first 30° north of the equator. The constellation Taurus was also in the second 30° north, and so on; the constellation and similarly named sign *then* occupied the same space of the heavens. Since that time, however, the signs have gone backwards among the constellations, that is, from east to west, contrary to the course of the sun; or the constellations have moved forwards, and now the change of place amounts to a complete sign, so that —

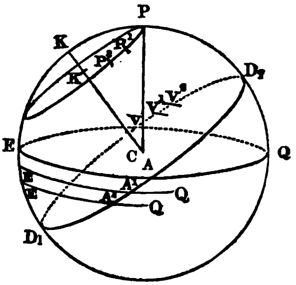
In sign Aries is the constellation Pisces;
" Taurus " " Aries;
" Gemini " " Taurus, &c. &c.

The spring constellations are Pisces, Aries, Taurus; but the spring signs are Aries, Taurus, Gemini. The summer constellations are Gemini, Cancer, Leo; but the summer signs are Cancer, Leo, Virgo, &c. &c.

Precession of the equinoxes.

(42.) This backward motion of the equinoctial points is called the precession of the equinoxes. It amounts to about 50·1 seconds in a year, so that the equinoctial points complete the circle of the ecliptic in a period of about 25,900 years. As the equator thus appears to move about within the ecliptic in this period, there must be a corresponding change in the relations of the axes of these two circles. The axis of the celestial equator, therefore, revolves also in that period round the axis of the ecliptic; and the polar points, which are the extremities of that axis, will not, therefore, be for ever near the same stars as previously described, but depart from them by slow degrees, and approach towards others, which will in their turn become the polar stars (8). In fig. 10 let the circle EPQ represent the celestial sphere, EAQV the equinoctial, D_1 AD_2 V the ecliptic, P the celestial pole of the northern hemisphere, that is, the pole of the equinoctial, K the pole of the ecliptic, V the autumnal equinoctial point, A the vernal; then, as the equinoctial points by precession assume the positions A, A_1, A_2, and V, V_1, V_2, in their progress round the ecliptic, the axis of the celestial sphere will assume the different positions PC, P_1C, P_2C, and the polar point will describe a circle round K in the same period as the equinoctial points revolve round the ecliptic, namely 25,900 years. By this revolution of the axis, the present pole star will lose its position as our cynosure, although very slowly; and, after the lapse of ages, the bright star in the constellation of the Lyre will be that nearest the pole of the heavens.

Fig. 10.



(43.) Imaginary circles in the celestial sphere, parallel to the horizon, are called circles of altitude, or sometimes Almacantar circles. Those parallel to the equator are specially called 'parallels,' while those parallel to the ecliptic have no distinguishing name.

The zodiac.

(44.) Of the circles parallel to the ecliptic two may be noted; namely, those which bound the space in the heavens, called the zodiac. These are circles, one on each side of the ecliptic, at a distance from it of about 10°. In the space between these circles the planets move. The name zodiac (from *ζοον*, animal) is given to it, because the twelve constellations contained in it are nearly all named from animals, as the Ram, the Bull, &c. &c.

North and south tropical circles.

(45.) Among the circles parallel to the equator, the most important are: —

1. The north and south tropical circles. The first of these passes through the northern solstitial point, the second through the southern; or, rather, the northern solstitial point in its daily motion describes the first, and the southern the second. Because the first passes through the beginning of the sign Cancer, it is called the tropic of Cancer; for a similar reason the second is called the tropic of Capricorn.

North and south polar circles.

2. The north and south polar circles.

Of these the first passes through the northern, and the second through the southern pole of the ecliptic; or, rather, the north pole of the ecliptic in the one case, and the south pole of the ecliptic in the other, during their daily motion, describe the respective circles.

The zones.

(46.) By the tropical and polar circles the surface of each of the hemispheres north and south of the equator is divided into three parts, called zones. The first lies between the equator and the tropical circles, the second between the tropical and polar circles; the third is the remainder of each hemisphere. It is easy to determine the breadth of each zone in degrees. That of the first is equal to the obliquity of the ecliptic ($23\frac{1}{2}°$); that of the third is the same, since the poles of the ecliptic and equator must be the same number of degrees apart as the ecliptic is oblique to the equator; hence there remains of the whole 90° from the equator to the pole, for the second or middle zone, $90-2 \times 23\frac{1}{2}° = 43°$.

Determination of the position of a

(47.) The position of a star in the heavens is described by showing how it stands in reference to the horizon, the equator, or the ecliptic. For this purpose two things must be given: — star on the celestial sphere.

1. The distance of the star from one of these circles measured upon another great circle passing through the poles of the first, and therefore perpendicular to it.

2. The points of these circles at which the perpendicular circle intersects them.

(48.) The great circle which passes through the poles of the horizon, namely, the zenith and the nadir, and also through the place of a star is called its vertical circle. The arc of this circle between the horizon and the place of the star is called its altitude. **Altitude.**

(49.) The great circle which passes through the poles of the equator, namely the North and South Poles, and the place of a star is called its circle of declination. The arc of this circle between the equator and the place of the star is called its declination. **Declination.**

(50.) The great circle which passes through the poles of the ecliptic and the place of a star is called its circle of latitude. The arc of this circle between the ecliptic and the place of the star is called its latitude. **Latitude.**

The altitude, declination, and latitude, are reckoned from 0 to 90°. The two last express distance either north or south of their respective circles. The complement of the altitude, or difference between it and 90°, gives the zenith distance of a star. The complement of the declination gives its polar distance. **Zenith distance.**

(51.) As the meridian goes through the zenith of a place, and is perpendicular to the horizon, it is also a vertical circle. The arc of the horizon between the vertical circle of a star and the meridian, or the angle formed at the zenith by the meridian and the vertical circle of the star is called the azimuth. It is reckoned, beginning from the south point and measuring in an **Azimuth.**

east and west direction on the horizon, from 0 to 180°, so that azimuth is either east or west.

Right ascension and longitude.

(52.) The point of greatest importance in referring a star to the equator or the ecliptic, is one of the points of intersection of these two circles, namely, the vernal equinoctial point. The arc of the equator between this point and the declination circle is called the right ascension; and the arc of the ecliptic between the circle of latitude and this point is called the longitude of the star. The right ascension and the longitude are counted on their appropriate circles from the vernal equinoctial point onwards from 0 to 360° in the direction of the signs; that is, like the annual motion of the sun among the stars from west to east. The position of a star in reference to the horizon is thus determined by altitude and azimuth: in reference to the equator by declination and right ascension, and in reference to the ecliptic by latitude and longitude.

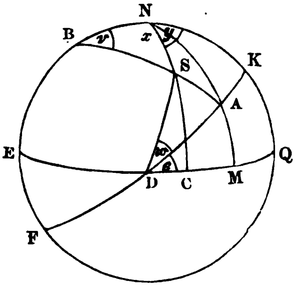
Deductions.

(53.) All stars in the same circle of altitude have the same altitude but a different azimuth; so all stars in the same parallel have equal declination but different right ascension; and all stars in the same circle parallel to the ecliptic have equal latitudes but different longitudes. The sun has no latitude, being always in the ecliptic. At the vernal equinox it has no longitude. At the northern solstice its longitude is 90°, at the autumnal equinox 180°, at the southern solstice 270°. At the same points in order, its right ascension is 0, 90°, 180°, 270°; and its declination 0, 23½° N., 0, 23½° S. The sun, when rising and setting, has no altitude; at midday it is at its greatest altitude. On March 21 and September 23 (on which days it rises exactly in the east point and sets in the west), at rising its azimuth is 90° east, and at setting 90° west.

To find latitude and longitude by trigonometry.

(54.) The right ascension and declination of a star are more easily observed than the latitude and longitude; but having obtained the first two by observation, the other two may be found by the rules of spherical trigonometry.

Fig. 11.



In fig. 11, EQ represents the equator, N the North Pole, FK the ecliptic, B its pole, NC is the declination circle of the star S, its latitude, measured on the circle BA, is AS, its declination CS, and right ascension DC. The last two being known, then $NS = 90° - CS$, and $CQ = 90° - DC$; and in the spherical triangle BNS there are known $BN = 23\frac{1}{2}°$; NS, the complement of the declination; and the included angle BNS, which is measured by $ED + DC = 90° +$ right ascension.

$$\therefore \cos BS = \cos BN \,.\, \cos NS + \sin BN \,.\, \sin NS \,.\, \cos BNS.$$

Let the longitude be l, the latitude b, the right ascension r, the declination d, the distance BN $(23\frac{1}{2}°)$ e, and let the angle SBN be v, BNS be x, and $SNK = (180 - BNS)$ be y, then we have from the above $\cos(90 - b) = \cos e \times \cos(90° - d) - \sin e \times \sin(90° - d) \times \cos y$; then, since $\cos y = \cos CQ = \cos(90 - r) = \sin r$, the expression for the latitude AS or b becomes $\sin b = \cos e \times \sin d - \sin e \times \cos d \times \sin r$.

To find an expression for the longitude corresponding to this latitude, we have in the same triangle

$$\sin NS : \sin BS = \sin v : \sin x, \text{ whence}$$
$$\cos SC : \cos SA = \sin AK : \sin CQ.$$

$\therefore \cos d : \cos b = \cos l : \cos r$, therefore the expression for the longitude becomes

$$\cos l = \frac{\cos d \times \cos r}{\cos b}.$$

To find the declination.

To find the declination of a star from its longitude and latitude, draw the arc of a great circle through S and D, then —

$\tan$ SDA or $\tan w = \dfrac{\tan b}{\sin l}$, thus w is known.

$\sin$ SD $= \dfrac{\sin d}{\sin (w+e)}$ from the spher. tri. SDC,

and $\sin$ SD $= \dfrac{\sin b}{\sin w}$ from the spher. tri. SDA.

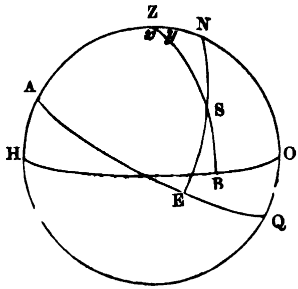
$$\therefore \frac{\sin d}{\sin (w+e)} = \frac{\sin b}{\sin w}, \text{ hence}$$

$$\sin d = \frac{\sin b \times \sin (w+e)}{\sin w}.$$

The sun has no latitude, for it is always on the ecliptic. Consequently $b=0$, and therefore the declination of the sun is one side of a right-angled triangle, whose hypotenuse is its longitude, and the third side is its right ascension. The angle subtended by the declination is the obliquity of the ecliptic. Suppose the sun in A, fig. 11, then this triangle is ADM, which gives $\sin$ AM $= \sin$ DA . $\sin$ ADM, that is,

$\sin d = \sin l \times \sin e$ ($e = 23\frac{1}{2}°$).

Fig. 12.



The determination of the declination of a star, when the altitude, azimuth and polar height are given, is not difficult. In fig. 12, HO is the horizon, AEQ the equator, Z the zenith, N the pole, ON the polar height (p), and S the star. BS is the altitude (h), HB or x the azimuth (a), and SE the required declination (d). In the spher. tri. ZNS,

ZS$=90° - h$, $y = 180 - a$, and ZN$= 90 - p$, hence

$\cos$ NS $= \cos$ ZN . $\cos$ ZS $+ \sin$ ZN . $\sin$ ZS . $\cos y$, or

$\sin d = \sin p \times \sin h - \cos p \times \cos h \times \cos a$.

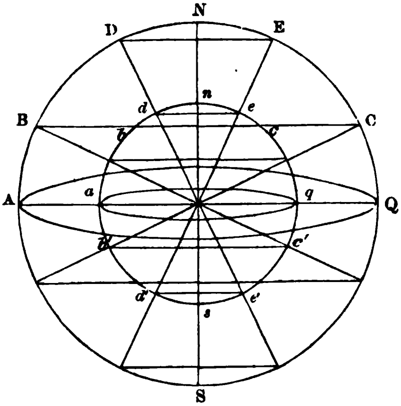
CHAPTER IV.

POINTS, CIRCLES, AND POSITIONS, ON THE SURFACE OF THE EARTH.

(55.) As the spheres of the earth and heavens are concentric, a great circle may be imagined on the earth's convex surface, corresponding to every great circle on the concave surface of the celestial sphere — in the same plane and concentric with it — having for its axis the part of the axis of the celestial sphere which passes through the earth. In fig. 13, AQ represents the celestial equator, *aq* the terrestrial, NS the axis of the celestial sphere, *ns* that of the earth, N and S the celestial poles, *n* and *s* the terrestrial, ABDNECQ the

The celestial and terrestrial spheres correspond.

Fig. 13.



northern half of the celestial sphere, *abdnecq* the northern half of the terrestrial, &c.

The places of the circles upon the earth's surface corresponding to the more important of the small circles in the celestial sphere may be found by describing circles at the same angular distances from the great circles with which these small ones are connected. Thus, if we make *cq* like CQ equal to 23½°, *ec* like EC = 43°, and *ne* like NE = 23½°, the places of the northern tropical and arctic circles are found, and the same circles in the southern hemisphere may similarly be found. Each of the hemispheres into which the earth is divided by the equator, is thus again divided by these circles into zones corresponding to those on the surface of the celestial sphere. The relation of the small circles upon the earth to the corresponding circles of the celestial sphere, may perhaps be best represented to the mind by imagining a cone, the base of which is one of these circles on the celestial sphere, and its vertex the centre of the earth. The intersection of the earth's surface with the slanting surface of the cone thus imagined gives the position of these circles on the terrestrial sphere. This is evident from fig. 13.

Terrestrial equator and meridian defined.

(56.) For every celestial meridian there is a corresponding meridian upon the earth's surface. As the one passes through the celestial poles and the zenith of a place, so the other passes through the terrestrial poles and the place itself; and we can now say that the terrestrial equator is that great circle of the earth which divides it into the northern and southern hemispheres; and a terrestrial meridian is a great circle of the earth which divides it into eastern and western hemispheres. There is but one equator — the meridians are many.

(57.) To determine exactly the position of a place upon the earth, requires the solution of three important problems, which can only be done satisfactorily by the aid of careful astronomical observations. These three are:—

Position on the earth is determined by:—

1. To determine the direction of a meridian line, or of the plane of the meridian.

2. To determine how far from the equator along a meridian line the place lies.

the latitude:

3. To determine how far from a certain meridian along the equator the meridian of the place lies.

and the longitude,

The second of these problems is called determining the latitude; the third determining the longitude. It is evident that latitude may extend from 0 to 90°, since a place may be 90° but not farther on the earth from the equator.

Longitude is sometimes reckoned from the meridian which is chosen as the first, half round the globe in an east and west direction, counting from 0 to 180° either way; sometimes it is reckoned all round the globe in an eastern direction from 0 to 360°. The first mode is called nautical longitude, the second geographical.

counted from a certain meridian.

(58.) The accuracy of all observations for obtaining the latitude and longitude of a place depends upon the accurate determination of the plane of the meridian.

Determination of the direction of the meridian:

For this purpose the observer may watch the sun or any other star, and these either at the time of culmination, or at times of equal altitude before and after their culmination.

a. When the sun culminates it is already on the meridian, and the shadow of a perpendicular staff upon a horizontal plane will, therefore, mark the direction of a meridian line.

by the sun's culmination;

b. The sun at equal heights before and after culmination, throws shadows equal in length, but towards different sides from the same symmetrical objects. If,

and by observations of shadows from a staff.

then, a staff be set perpendicularly on a perfectly level plane, and round it on the plane, there be drawn a number of concentric circles, the line bisecting the angle formed by two shadows of equal length will be the meridian line. Two equally long shadows show periods of the day equally distant from the instant of noon; for example the shadow at 10 A.M. will be equal to that at 2 P.M., and that at 11 A.M. will be equal to that at 1 P.M.

By a sextant, and chronometer.

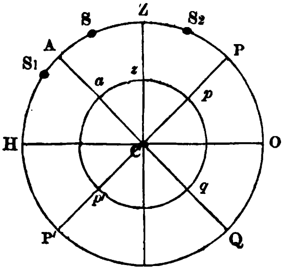
c. The above method of finding the meridian is imperfect, because the termination of the shadow of the staff is always indistinct. To find its direction exactly a telescope, a chronometer, and a sextant are required. The sextant enables us to find two moments, one before and one after noon, in which a star or the sun has the same altitude; the chronometer gives the interval between these two moments; half this interval gives the time between the first moment of observation and the moment when the star crossed the meridian. If we add this interval to the time when it had attained the first altitude, we can determine by the chronometer the instant at which, on the next day, it will cross the meridian. A telescope so directed that the star will be in its axis at that instant next day must necessarily be in the direction of the meridian, and if it be fixed in this position to a horizontal axis perpendicular to this direction, so that it may have a vertical motion, this motion will be in the plane of the meridian. Such an instrument is called a transit instrument; by it the exact instant when any celestial body crosses the meridian is determined.

The transit instrument.

The latitude equals the elevation of the pole.

(59.) To determine the latitude. The latitude of a place is equal to the elevation of the pole above the horizon of the place. In fig. 14, the smaller circle represents the earth, the larger the celestial sphere; HO

Fig. 14.



the horizon of the place *z*, Z its zenith; AQ the celestial and *aq* the terrestrial equator; PP the axis of the celestial sphere, *pp* that of the earth; P and P the celestial north and south poles respectively, *p* and *p* the north and south poles of the earth. The height of the north pole at Z is OP; AZ in the celestial sphere, and *az* in the terrestrial, represent the latitude, since the first is the distance on the celestial meridian between the zenith of the place and the equinoctial, and the other is the distance on the earth itself, of the place from the equator. Then, because the distance of the zenith from the horizon is equal to the distance of the pole from the equator, each being one fourth of the circle, we have $ZO = AP$ and $ZO - ZP = AP - ZP$, therefore $OP = AZ = az$, that is, the height of the pole is equal to the latitude. By observing the one, therefore, we know the other. The altitude of the equator above the horizon, namely, the arc HA, is the complement of AZ; it is, therefore, also the complement of the polar height, that is the difference between the latitude and 90°. Hence having given the latitude we may easily find the altitude of the equator and vice versâ.

The altitude of the equator is the complement of the latitude.

(60.) To determine the elevation of the pole, advantage is taken of the fact that, in its daily circle every star passes the meridian twice, at equal distances from the pole and on opposite sides of it. For direct observation of the polar height, a star that never sets in our latitude is usually chosen — it may be the polestar itself. By means of a quadrant its exact height is

Determination of the polarheight.

noted at the instant of upper culmination, and again at that of lower. It is evident that the actual pole of the heavens must be in the central point of the star's path, and therefore exactly in the centre of these two culminating points. Half the sum therefore of the greatest and least altitudes will give the true altitude of the pole.

(61.) Instead of the greatest and least altitudes of a star, its greatest and least zenith distances at the instant of culmination may be found, and then half their sum will be the complement of the polar height or latitude. E.g. at Dublin the greatest zenith distance of
the polestar was found to be . . . 38° 19′ 43·11″
Its least distance was . . . . . . 34° 53′ 49·55″

Their sum . . . . 73° 13′ 32·66″

Half this, or the colatitude of Dublin,
and altitude of the equator there . 36° 36′ 46·33″
which subtracted from 90° gives the
latitude of Dublin . . . . 53° 23′ 13·67″

Latitude found by zenith distance of a star.

(62.) By observing the zenith distance of a star or the sun when it passes the meridian, and knowing its declination, we can at once ascertain the distance of the zenith from the equinoctial, that is, the latitude. Let S, S_1, S_2, fig. 14, be different positions of the star to be observed, which, for instance, may be the sun. The declinations of the sun for these different positions will be AS, AS_2 north of the equator, and AS_1, south of it. Let l be the latitude required, d the declination, z the zenith distance; then we have $l = \text{ZA} = \text{ZS} + \text{AS}$ or $\text{ZS}_1 - \text{AS}_1$ or $l = z \pm d$. That is, the latitude is equal to the meridional zenith distance of the sun, plus or minus the declination according as the sun at the place of observation is above or below the equator. The zenith distance is generally found by observation of the altitude and subtracting it from 90°.

(63.) The navigator requires to determine his latitude at sea daily. He finds, by means of a sextant, the sun's altitude at the instant it passes the meridian. From this he deduces his zenith distance. The Nautical Almanac tells him the sun's declination, or distance from the equator, on the day of observation. The sum or difference of these is his latitude, according as the sun and the ship are on the same or different sides of the equator. Suppose that he is north of the equator on a day when the almanac tells him that the sun's declination is 20° north.

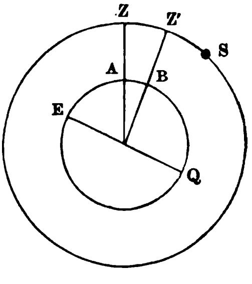
Let him find the sun's altitude at noon to be	59° 0 0
He will subtract that from	90° 0 0
Which gives him the zenith distance . . .	31° 0 0
He then *adds* the declination	20° 0 0
And finds his latitude to be, north. . . .	51° 0 0

It is possible that the observation may require to be made when the star or sun is north of the zenith, as at S_2. This would be the case if the vessels were *within* the tropics and the sun at its greatest northern declination of 23½°.

Then $l = ZA = AS_2 - ZS_2 = d - z$.

(64.) When the latitude of one place is known, that of another may be found by observing with a sextant the zenith distances of any known star at both places. The difference of the two zenith distances will give the difference of latitude at the two places. In fig. 15 let the outer circle represent the celestial sphere, the inner the terrestrial; then it is evident that the arc AB = arc ZZ′ in degrees.

Fig. 15.



Let A and B be two different places and EQ the equator, then the arc AB or ZZ′ is their difference of latitude. Let a star S be observable at the two places, then ZS is the zenith distance of S at A, and Z′S its zenith distance at B. But $ZS - Z'S = ZZ' = AB$, the difference of the latitudes.

The zenith distance of the star β of the constellation Draco at Paris was	3°	37′	2·63″
And at Greenwich it was	0°	58′	33·13″
The difference of latitude therefore .	2°	38′	29·5″
The latitude of Greenwich is known to be	51°	28′	40·5″
Therefore the latitude of Paris is . .	48°	50′	11″

Determination of longitude.

From the uniform apparent motion of the sun:

(65.) To determine longitude. If we suppose a number of meridians drawn upon the sphere of the earth, so that they intersect at the poles, making equal angles with each other, thus also crossing the equator at right angles and at equal distances measured on the arc of that great circle, it is evident that, as the sun or a star passes over each of these meridians once every twenty-four hours, it must pass over the whole circumference of the earth in that time. These twenty-four hours of time, therefore, must correspond to 360° of longitude; and as the sun moves from east to west, it will necessarily pass over the meridian of places to the east, and bring noon to them earlier than it does to places farther west. As it passes over all the meridians in the mean interval of twenty-four hours, with a uniform rate of apparent motion, we are thus afforded a means of ascertaining how far in the east or west direction the meridian of one place is situated upon the globe from the meridian of another. Since the sun passes over 360° in twenty-four hours, we can easily calculate that he must pass over 15° in one hour; nor will this calculation be in any way vitiated, although it should be shown afterwards (as it will) that it is not the sun which passes over all the meridians, but all the meridians which pass under the sun in twenty-four hours. Therefore, when it is noon at Greenwich, the sun will have passed over the meridian of a place 15° eastward of Greenwich one hour before, and it will now be 1 o'clock P.M. at that place. So, one hour must yet elapse before he brings noon to a place whose meridian lies 15° westward, and it will, therefore, be only 11 o'clock A.M. there. And, generally, if we choose the meridian which passes through Greenwich as the first from which we shall begin to reckon the distance or longitude of all other meridians, the difference between the time at the meridian of Greenwich and that of all other meridians will be found by expressing the longitude in time, allowing one hour for every 15°, and other parts proportionately; and remembering that the time is in *advance of* or *behind* that of Greenwich, according as the longitude is east or west from Greenwich.

and difference of local time.

(66.) It follows from this that, if we can by any means learn the hour of the day at two places on different meridians, we may tell their difference of longitude; and if we choose to consider one of these the first meridian, the actual longitude of the other will be known. The determination of the longitude of a place thus resolves itself into the finding of the difference of time between a place on the first meridian and that of a place whose longitude is required; then comparing these times, and allowing a proportion of 15° to every hour of difference, *eastward* if the time at the place be in advance of that of the first meridian, *westward* if it be behind.

Three general methods of comparing time.

(67.) There are different methods of ascertaining at one place the time of the day at another.

1. By conveying correct timepieces or chronometers from the one place to the other.

2. By observation of certain phenomena upon the earth, which are perceptible at both places at the same *absolute* instant of time, and comparing afterwards the two local times at which the phenomena took place.

3. By observation of similar phenomena in the heavens.

First, by chronon e-ters.

(68.) Suppose a traveller to leave Greenwich, carrying with him a correctly-going watch set to the mean solar time there, and showing, therefore, 12 o'clock at the instant of mean noon at Greenwich. Let him go to Edinburgh and compare his watch with those set to the proper local time there; he will find that his watch is before these. At twelve by Edinburgh time, his watch will show 12 minutes, 44 seconds past twelve. He may, therefore, conclude that he is west from Greenwich as many degrees as corresponds to that difference of time, namely 3° 11′. Since

$$1 \text{ hour} : 12 \text{ min. } 44 \text{ sec.} = 15^\circ : 3^\circ\, 11' =$$

long. of Edinburgh. Or, again, let our traveller, on leaving Greenwich, proceed to Paris. Here he finds that his watch is behind by 9 minutes, 21 seconds, which gives a longitude east of Greenwich of 2° 20′.

The navigator determines his longitude at sea by chronometers on the same principle. He carefully ascertains by his sextant the instant of the sun's crossing his meridian, applies certain corrections so as to have the very instant of mean noon at the place where he is, and then compares that with Greenwich time, which his chronometer, if good, shows.

Second, by observations of terrestrial phenomena.

(69.) For places not very far from each other, objects visible at both places in the same absolute instant suffice to show the difference of the local time.

Ascent of rockets.

Let us suppose that a rocket is fired at some central spot, and visible from the two places whose difference of longitude is to be ascertained. Let A and B, fig. 16, be the two places, and C that from which the rocket is sent up. The instant at which it rises is noted by observers at A and B; the time on the clocks at the two places is also noted, and thus the means of comparing their difference of time is obtained. If the two places be so distant that one rocket cannot be seen from both, a number of rockets sent up from different intermediate places will serve the purpose. One sent up between the places A and C, fig. 17, will afford a comparison of the local times of these two places; a second sent up between C and D will show the difference of their times; so between D and E, and E and B. The sum of all these partial results will give the difference between the clocks at A and B, and hence their difference of longitude may be deduced. Flashes of gunpowder may be similarly used, or brilliant lights such as the Drummond light exhibited from mountain tops; e.g., the difference of longitude between Dublin and Edinburgh might, doubtless, be accurately ascertained by means of such a light on Ben Lomond—the first flash when it is lighted up, or its sudden extinction, being visible from both places at the same absolute instant of time.

Fig. 16.

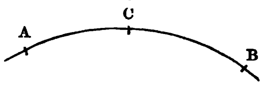
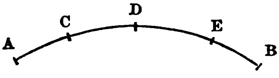


Fig. 17.



Sudden lights.

Electric telegraph.

(70.) The modern invention of the electric telegraph affords a means of comparing the simultaneous indications of two clocks at different places connected by telegraphic wires. A signal from the one end transmitted instantaneously to the other may be observed by some one there, and thus the times at the two extremes compared.

Third, by observations of celestial phenomena.

(71.) But for places not connected by telegraphic wires, and too distant for the method of rocket or other signals, recourse may be had to some of the celestial phenomena which are generally visible over a whole hemisphere at the same absolute instant of time.

Eclipses of Jupiter's satellites.

a. The eclipses of Jupiter's satellites are phenomena of this kind. The entrance of each satellite into the part of space where Jupiter's shadow is thrown may be seen almost at the same absolute instant of time over the whole hemisphere in which Jupiter is visible, and the times at which such eclipses will happen can be predicted with exactness. These are computed for Greenwich time, and set down in the Nautical Almanac for several years before they happen. The means is thus afforded the traveller of comparing his local time with that of Greenwich, and thus ascertaining his longitude. This method cannot be used at sea, because the rolling of the vessel renders it impossible to direct a telescope towards Jupiter so steadily as to view an eclipse of a satellite.

Eclipses of the moon.

b. Eclipses of the moon, to be hereafter explained, afford another means of obtaining the longitude. Such an eclipse has exactly the same appearance wherever it is visible. The instant of the immersion of the moon in the earth's shadow (194) may be noted by different observers at different places, and also the instant of its emergence from the shadow. There are thus two independent phenomena, by means of which the local times may be compared and made to correct each other's error; e.g.—

An observer at Paris sees the immersion at	$12^h\ 19^m\ 13^s$
An observer at Barbadoes sees the immersion at	$8^h\ 11^m\ 0^s$
The difference of their time is therefore	$4^h\ 8^m\ 13^s$
Again, he at Paris sees the emergence from the earth's shadow at 59 minutes past one o'clock . . .	$1^h\ 59^m\ 0^s$
He at Barbadoes at	$9^h\ 51^m\ 0^s$
Their difference of time by this second observation is	$4^h\ 8^m\ 0^s$

The mean of the two observations may be taken, which gives for the difference of the time of the two places 4 hrs. 8 m. 6·5′, and a consequent difference of longitude of 62° 1′ 30″, which will be the longitude of Barbadoes, taking the meridian of Paris as the first.

Eclipses of the sun.

c. Eclipses of the sun (192) may be similarly used.

Meridian passage of the moon.

(72.) The passage of the moon over the meridian is another method by which longitude may be found.

It has already been observed (10) that the sun's motion among the stars is slower than that of the moon; if, therefore, they pass together over the meridian of any place, as Greenwich, on one day, they will not pass together on the next; but as the moon will be farther east than the sun, it will cross the meridian somewhat later. The time which thus occurs of delay in the moon's passing the meridian is called the moon's retardation. It is obvious that this time will increase constantly and gradually, so that if the retardation be a certain amount when the moon passes the meridian of

Greenwich, it will be something more, when, later in the same revolution it comes to pass over the meridian of a place *west* of Greenwich; and the farther west the place the greater will be the time of retardation. It is from this fact that we derive a means of ascertaining longitude. Let the sun and moon have passed the meridian of Greenwich together on a certain day, and let it be found that on the next day the moon's retardation at Greenwich is 52 minutes. Suppose now, that at a place west of Greenwich, the longitude of which is required, the retardation is found to be 57 minutes, that is, five more than at Greenwich. This five minutes is due to the western longitude. We may now put the question thus:—If the moon in moving through 360° before it reached the meridian of Greenwich was retarded 52 minutes, in how many degrees will it be retarded 5 minutes more? These degrees, when found, will give the longitude required. The proportion thus becomes:—$52' : 5' = 360° : 360° \times \frac{5}{52} = 34° \, 37'$. Hence generally the longitude of a place is equal to 360° multiplied by the difference between the moon's retardation at Greenwich and that at the given place, and divided by the increase of the moon's retardation at Greenwich during the 24 hours preceding the observation. The increase of retardation at a given place is known by observation; that at Greenwich for the previous 24 hours is to be found in the Nautical Almanac.

Lunar method.

(73.) The last of the methods of finding longitude to be here noticed is that called the method of lunar observations.

If it were possible to place a large clock in the heavens, the face of which should be visible from all parts of the earth, as the stars are, and which should be constantly kept by the local time of Greenwich or any other first meridian, then the means of finding our longitude would evidently always be at command. For we should only require to observe the time indicated on that clock, and might compare it at once with our local time. Astronomers have most ingeniously and industriously contrived to make such a clock of the whole face of the heavens; the stars serve the purpose of the hour divisions on an ordinary clock, and the moon moving according to known laws among the stars, gliding beside some and hiding others, is the hand which points to the hour divisions. Her exact position in reference to certain standard stars is calculated years beforehand, and noted in Greenwich time for every third hour of every day in the year. These results are published in the Nautical Almanac. The almanac may therefore be considered as the figures, enabling us to tell which hour division the hand is pointing at. As soon as an observer, in any part of the world, has measured the distance of the moon from any standard star, at a given instant of his local time, or has ascertained the exact instant of her hiding (or occulting, as it is called) any such star he may find his longitude: for in the almanac he will find the instant of that observed angular distance, or occultation, in Greenwich local time.

National reckonings of longitude.

(74.) It is unfortunate that all nations do not use the same first meridian, but each selects its own. Ptolemy fixed his at the Fortunate or Canary Islands; Louis XIII. of France ordered that Ferro, the most westerly of these islands, should be chosen, as it was then thought to be 20° west of Paris. It was afterwards, however, found to be 20° 5′ 50″ from that capital, so that the meridian now called that of Ferro passes through no remarkable place; Mercator chose one of the Azores, named Del Corvo, 31° 3′ west of Greenwich; the Venezuelans chose Caraccas, 66° 44′ 37″

W.; the Americans, Washington, 77° 2′ 1″ W.; the French, Paris, 2° 20′ 23″ E.; the English, Greenwich. Hence it often becomes necessary to convert the longitudes of one system of reckoning into those of another, e.g. Dover is 1° 18′ 30″ *east* from Greenwich: what is its longitude referred to the meridian of Paris?

Paris is 2° 20′ 23″ *east* of Greenwich, therefore Dover is 2° 20′ 23″ − 1° 18′ 30″ = 1° 1′ 53″ *west* from Paris.

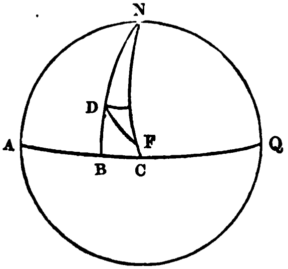
Tortoise Island is 177° 57′ *west* longitude from Greenwich, what is its longitude from Paris?

Paris being *east* from Greenwich, 177° 57′ + 2° 20′ 23″ = 180° 17′ 23″ is the longitude west from Paris; but this is more than half the equatorial circle; hence 360° − 180° 17′ 23″ will give the longitude *east* of Paris.

Difference of geographical and astronomical latitude and longitude.

Geographical latitude and longitude should be carefully distinguished from astronomical. Geography refers these measurements to the equator — astronomy to the ecliptic.

Fig. 18.



(75.) The calculation of the shortest distance between two places when their latitude and longitude are given is as follows:—

In fig. 18, if the longitudes AB and AC and the latitudes DB and FC of the two places D and F are given, then the angle BNC is given, since BC = AC — AB.

Again, NB = NC = 90°, therefore ND and NF are the complements of the given latitudes; hence, in the spherical triangle DNF we have

$$\cos DF = \cos ND \times \cos NF + \sin ND \times \sin NF \times \cos DNF,$$

and DF is the distance required.

(76.) The shortest distance, as DF, between two places on the globe is an arc of a great circle. This is evident by considering that the plane of a great circle passes through the centre of the globe, and that of a small circle does not. The radius of a small circle is less than that of a great one, and the curvature or bending, therefore, of a small circle is greater than the curvature of a great one. Now a straight line is the shortest possible distance between two points; and of two curved lines between these points that which is least curved must be the shorter. An arc, therefore, of a great circle between two points must be shorter than an arc of a small one; and on the surface of the earth all motion between two places must take place on the arc of some circle.

The shortest distance an arc of a great circle.

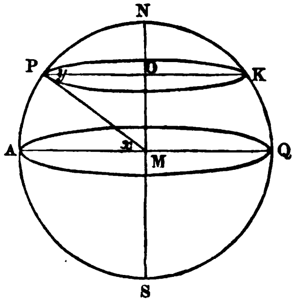
(77.) The length of a degree of latitude being everywhere measured upon a meridian, which is a great circle, is always the same, and equal to $\frac{1}{360}$th of the meridian. The lengths of degrees of longitude differ. Except on the equator they are nowhere measured on a great circle, and indeed are measured on parallel circles which constantly diminish in circumference as the latitude increases. The circumference of each parallel circle is always 360°, therefore the lengths of the degrees in each must diminish as they approach the poles.

Degrees of longitude decrease towards the poles.

(78.) The length of a degree of longitude varies as the cosine of the latitude. Hence, if we know the latitude (b) of any place and the radius of the earth (R), we may calculate the length of a degree of longitude in different parallels of latitude.

In fig. 19, let AQ represent the equator, PK a parallel circle passing through the place P, NASQ the meridian of that place, and NS the axis of the earth, PO (r) is parallel to AM (R). Draw from the centre M the radius MP, then the latitude PA (b) is the measure of the angle AMP (x) = MPO (y); and we have PO = PM cos y or $r = \mathrm{R} \cos b$. Also, for b' any other latitude we have $r' = \mathrm{R} \cos b'$, hence $r : r' = \cos b : \cos b'$, that is, the radii of different parallels of latitude are to each other as the cosines of the latitudes, and the circumferences will also be in the same ratio; and hence, too, the $\frac{1}{360}$th parts of these circumferences, or the degrees, are as the cosines of the latitudes. The equator itself may be considered as a parallel circle whose latitude is 0°, and cosine therefore = 1.

Fig. 19.



If we suppose the circumference of the earth at the equator to be about 24,897 miles, then a degree will be 69·16 miles, the circumference of a tropical circle will be $24897 \times \cos 23\frac{1}{2}°$, and the length of a degree on it $69{\cdot}16 \times \cos 23\frac{1}{2}°$. So, too, the circumference of a polar circle will be $24897 \times \cos 66\frac{1}{2}°$ in miles, and the length of a degree on it $69{\cdot}16 \times \cos 66\frac{1}{2}°$.

TABLE SHOWING THE LENGTH OF ONE DEGREE OF LONGITUDE FOR EVERY DEGREE OF LATITUDE IN GEOGRAPHICAL AND ENGLISH MILES.

Lat.	Geog. Miles	English Miles	Lat.	Geog. Miles	English Miles	Lat.	Geog. Miles	English Miles
0°	60·00	69·16	31°	51·43	59·13	61°	29·09	33·45
1	59·99	69·06	32	50·88	58·51	62	28·17	32·40
2	59·96	69·03	33	50·32	57·87	63	27·24	31·33
3	59·92	68·97	34	49·74	57·20	64	26·30	30·24
4	59·85	68·90	35	49·15	56·51	65	25·36	29·15
5	59·77	68·81	36	48·54	55·81	66	24·40	28·06
6	59·67	68·62	37	47·92	55·10	67	23·45	26·96
7	59·55	68·48	38	47·28	54·37	68	22·48	25·85
8	59·42	68·31	39	46·63	53·62	69	21·50	24·73
9	59·26	68·15	40	45·96	52·85	70	20·52	23·60
10	59·09	67·95	41	45·28	52·07	71	19·53	22·47
11	58·89	67·73	42	44·59	51·27	72	18·54	21·32
12	58·69	67·48	43	43·88	50·46	73	17·54	20·17
13	58·46	67·21	44	43·16	49·63	74	16·54	19·02
14	58·22	66·95	45	42·43	48·78	75	15·53	17·86
15	57·95	66·65	46	41·68	47·93	76	14·52	16·70
16	57·67	66·31	47	40·92	47·06	77	13·50	15·52
17	57·38	65·98	48	40·15	46·16	78	12·48	14·35
18	57·06	65·62	49	39·36	45·06	79	11·45	13·17
19	56·73	65·24	50	38·57	44·35	80	10·42	11·98
20	56·38	64·84	51	37·76	43·42	81	9·38	10·79
21	56·01	64·42	52	36·94	42·48	82	8·35	9·59
22	55·63	63·97	53	36·11	41·53	83	7·31	8·41
23	55·23	63·51	54	35·27	40·56	84	6·27	7·21
24	54·81	63·03	55	34·41	39·58	85	5·22	6·00
25	54·38	62·53	56	33·53	38·58	86	4·18	4·81
26	53·93	62·02	57	32·68	37·58	87	3·14	3·61
27	53·46	61·48	58	31·79	36·57	88	2·09	2·41
28	52·97	60·93	59	30·90	35·54	89	1·05	1·21
29	52·48	60·35	60	30·00	34·50	90	0·00	0·00
30	51·96	59·75						

CHAPTER V.

THE MAGNITUDE OF THE EARTH.

Measurement of the earth.

(79.) ASSUMING the earth to be a sphere, could w descend into its bowels and measure the radius, i would be easy to tell its circumference, surface, &c. for the radius being R, its circumference would b $2R\pi$, its surface $4R^2\pi$, and its solid content $\frac{4R^3\pi}{3}$, &c.

Or, were the polar ices not in the way, and other hindrances to keeping exactly on a meridian line removed, we might travel round the circumference of the globe and measure it; but either of the above methods is impossible. Fortunately, however, for the solution of the problem of the earth's magnitude, the measurement of a *whole* meridian is unnecessary; that of a small part suffices.

By measuring an arc on its surface.

(80.) It has been shown (64) that the difference of the zenith distances of a star gives the difference of latitude between the two places situated on the same meridian. It is evident, therefore, that if we can find two places on the same meridian where a known star stands one degree farther from the zenith at one place than at the other, and measure the distance on the earth between these places, we must have the length in miles, &c. of one degree of the earth's circle. This length, multiplied by 360, will give the whole circumference of the earth.

The greater the number of places on the same meridian, whose latitudes we can observe and distances measure, the more likely we are to judge of the true length of a degree, and the more correctly we will be enabled to judge whether or not the earth be a perfect sphere. We shall see by and by that, tried by this test, the sphericity of the earth is not perfect.

(81.) To measure on the earth's surface any arc of a meridian, the number of degrees in which we know, by the comparative zenith distances of some given stars at the extreme points of that arc, is one of the most important operations connected with geographical science. It is also one of the most difficult; for there are but few places on the earth where it is possible to move far, and keep exactly in the direction of a meridian. Inequalities of the surface interfere; rivers intervene; forests, marshes, mountains impede the surveyor's progress, and compel him, instead of actual measurements by the rod or chain, to adopt a system of measurement of triangular or other figures formed by straight lines joining accessible points on one or both sides of the meridian, the length of whose arc is required.

Method of measuring an arc.

(82.) In 1768 the astronomers, Mason and Dixon, succeeded in measuring an arc of the meridian 538,078·39 English feet in length, along the boundary line between the States of Maryland and Virginia, through the peninsula formed by the Bay of Chesapeake and the mouths of the rivers Potomac and Delaware.

Fig. 20.

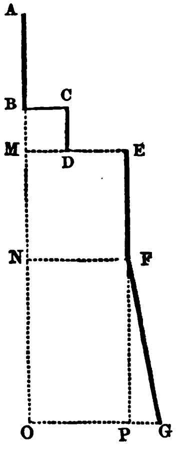
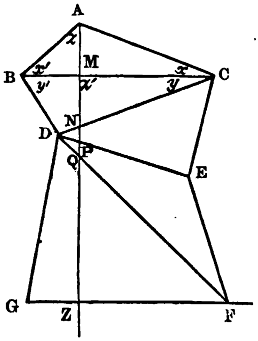


Fig. 20 will show the method by which this was effected. Four lines were measured, three of them along meridian lines, but not along the same meridian. The point A was on the meridian to be measured. The point B was so chosen that its latitude was the same as that of C; D the same as that of E. These three lines being measured directly, and their lengths added, gave the length of the original meridian from A to F′ on the same parallel of latitude as N. In the fourth line the surveyors were compelled to deviate from the direction of the meridian EFP; but they could measure the angle which their new line formed with that meridian; and, having measured FG, they were enabled, by trigonometrical calculation, to find the line FP. (FP = FG cos PFG.) This, added to the former result, gave the whole line AO. The line thus measured extended from latitude N. 39° 56′ 19″ to 38° 27′ 34″, passing thus over 1° 28′ 45″. By the proportion 1° 28′ 45″ : 1°= 538078·39 ft. : x, we have 363,786 feet as the length of an arc of 1° in these latitudes. This is not quite 69 English miles.

(83.) But it is not often that an arc of the meridian is so easily measured. More often it is impossible by actual measurement to proceed very far in the direction of a meridian; and a series of triangles is constructed about the meridian, beginning from a *carefully measured base line.* For instance, let it be required to find the length of a part of a meridian, represented by AZ, fig. 21. Along this meridian let it be impossible to proceed. Then points A, B, C, D, E, F, &c., are chosen on each side of the meridian, on which are conspicuous objects, such as elevated towers, steeples, hills with poles, trees, &c. Let the points in which these objects are situated be joined by straight lines, so

Fig. 21.



as to form a series of triangles across the meridian AZ. Let the side AB be chosen as the base line, on account of its being in a favorable position for being *exactly* measured, that is, on level ground, and attainable at all its points, &c. All that is necessary now is, not to measure the other sides of the triangles, but only all the angles of the triangles, and also the angle BAM (z), formed by the meridian AZ, with the accurately measured base AB. The sides of the triangles would then be found as follows (naming the angles by the small letters):—

$$\text{AB} : \text{AC} = \sin x : \sin x', \text{ and}$$
$$\text{AB} : \text{BC} = \sin x : \sin (x + x'),$$

hence the sides AC and BC are known, since

$$\text{AC} = \frac{\text{AB} \sin x'}{\sin x}, \text{ and } \text{BC} = \frac{\text{AB} \sin (x + x')}{\sin x}.$$

Again, in the triangle BDC,

$$\text{DC} = \frac{\text{BC} \sin y'}{\sin (y + y')} = \frac{\text{AB} \sin (x + x') \sin y'}{\sin x \,.\, \sin (y + y')}, \text{ \&c. \&c.}$$

Knowing now the sides and angles of the triangles, and also the angle z (BAM), we can, by a similar process, find the length of the part AM of the meridian. For, in the triangle ABM, we know the side AB, and the two adjacent angles z, x', and

$$\text{AM} : \text{AB} = \sin x' : \sin (z + x'), \text{ therefore}$$
$$\text{AM} = \frac{\text{AB} \sin x'}{\sin (z + x')} : \text{ similarly } \text{BM} = \frac{\text{AB} \sin z}{\sin (z + x')},$$

and the angle $\text{BMA} = 180° - (x' + z) = z'$, the alternate angle.

Again, in the triangle MCN, we know the side CM, which is the difference between BC and BM, and the two adjacent angles MCN and CMN (y and z'), hence we may find MN, the second part of the meridian, as well as the side CN, and the angle CNM.

$$\text{MN} : \text{MC} : \sin y : \sin (y + z') \therefore$$
$$\text{MN} = \frac{\text{MC} \sin y}{\sin (y + z')}.$$

So from the triangle DNP we know the third portion NP of the meridian, and continuing thus, we arrive at the length of all the different parts, which we may add for the whole length.

(84.) In the process of triangulation, the height of the points A,B,C,D,E,F, &c., fig. 21, above the level of the sea must be allowed for, and the triangles reduced, as it is called, to the horizon. This reduction being made, the angles and sides now found are the angles and sides of spherical triangles — that is, of triangles existing, not on a *plane*, but on the surface of a sphere; or, to speak more exactly, of an ellipsoid, as will afterwards be shown. In triangles whose sides do not exceed fifteen miles, the surface may, without practical error, be considered as a plane; in larger ones the true form must be taken into account. It may be noticed that, as the angles of a spherical triangle are always greater than two, and less than six right angles; and as the sum of the three observed angles of any of the great triangles is always found to be greater than two right angles, we have here another argument for the spherical form of the earth. Were the earth's surface a plane, the sum of these angles ought always to be exactly 180°. The triangles may be treated as plane triangles by deducting from each angle the third part of the excess of their sum above two right angles.

Another argument that the earth is round.

Results.

(85.) Having ascertained by such measurements that a degree of a great circle on the earth is 60 geographical or 69·16 English miles, we are enabled to make the following calculations: —

Circumference of earth $= 360 \times 69{\cdot}16 = 24{,}897$ miles.

Diameter $= \frac{24897}{3{\cdot}1416} = 7{,}924$ miles.

Radius = half diameter = 3,962 miles.

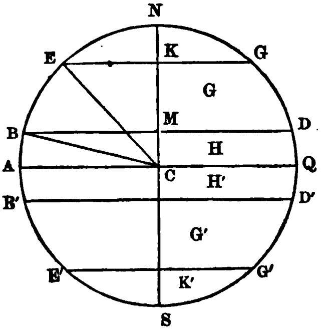
Surface $= 4\text{R}^2\pi = (\text{dia.})^2 \times \pi = (7924)^2 \times 3{\cdot}1416$.

Solid content $= \frac{4\text{R}^3\pi}{3} = \frac{(\text{dia.})^3\pi}{6} = \frac{(7924)^3 \times 3{\cdot}1416}{6}$.

Calculation of the surface of each zone:

(86.) In fig. 22, NS represents the axis of the earth; EG and E′G′, sections of the planes of the polar circles; BD and B′D′, similar sections of the tropical circles; AQ, of the equator. Let us call the zone between the equator and northern tropic, H; that between the northern tropic and the northern polar circle, G; and the end zone at the north pole, K. We may now calculate the surfaces of these zones for both hemispheres.

Fig. 22.



Spherical geometry shows that the surface of any spherical zone is equal to the product of the circumference of a great circle of the sphere and the height of the zone.

The circumference of a great circle $= 2\text{R}\,.\,\pi$, and the height of zone $\text{H} = \text{CM} = \text{R}\,.\,\sin 23\frac{1}{2}^\circ$

$\therefore$ area of $\text{H} = 2\text{R}\pi \times \text{R}\,.\,\sin 23\frac{1}{2}^\circ = 2\text{R}^2\pi\,.\,\sin 23\frac{1}{2}^\circ$.

And twice the area of H, or whole tropical zone $=$

$4\text{R}^2\pi\,.\,\sin 23\frac{1}{2}^\circ = (\text{dia.})^2\pi\,.\,\sin 23\frac{1}{2}^\circ = (7924)^2\pi\,.\,\sin 23\frac{1}{2}^\circ$.

Similarly, twice the area of F, or the surface of the temperate zones $=$

$$(7924)^2\pi \times (\sin 66\tfrac{1}{2}^\circ - \sin 23\tfrac{1}{2}^\circ).$$

And the area of the two end or frigid zones $= 2\text{K} =$

$$(7924)^2\pi \times (1 - \sin 66\tfrac{1}{2}^\circ).$$

And of the area of the globe.

So that $2\text{H} = (7924)^2 \times 3{\cdot}1416 \times \sin 23\frac{1}{2}^\circ$

$2\text{G} = (7924)^2 \times 3{\cdot}1416 \times (\sin 66\frac{1}{2}^\circ - \sin 23\frac{1}{2}^\circ)$

$2\text{K} = (7924)^2 \times 3{\cdot}1416 \times (1 - \sin 66\frac{1}{2}^\circ)$

$\therefore 2\text{H} + 2\text{G} + 2\text{K} = (7924)^2 \times 3{\cdot}1416 =$ area of the globe.

If it be required to find the area of a segment of any zone bounded on both sides by meridians, all that is necessary is to find first the area of the whole zone, and then to multiply the result by the number of degrees and parts of a degree in the length of the segment, and divide by 360; e.g.,

What is the area of a segment of the torrid zone between the meridians of 150° and 177° 30′ W.?

By the formula above, the area of the torrid zone will be found equal to 78,792,363 square miles; then

$$\frac{78792363 \times 22\frac{1}{2}}{360} = 4{,}924{,}522 \text{ square miles.}$$

Use of such calculations.

The States of the New World, instead of the so-called natural boundaries formed by rivers, mountain-chains, &c., have adopted the more unchanging boundaries formed by lines of latitude and longitude. Where this method exists, the use of such calculations as the above in finding the areas of different states is evident. E.g.—the State of Pennsylvania forms very nearly a quadrilateral between latitudes 39° 43′ 25″ and 42°, on the one hand, and longitudes 2° east of Washington and 3° 21′ 30″ west, on the other. The means of calculating its area may be found as follows:—

$42° - 39° 43' 25'' = 2° 16' 35''$ . . . height of the zone.

$2° + 3° 21' 30'' = 5° 21' 30''$ . . . length of the segment.

Then, as above—

$$\frac{(7924)^2 \times 3{\cdot}1416 \times \sin 2° 16' 35'' \times (5° 21' 30'')}{360°} = \text{area of}$$

Pennsylvania.

Irregularity on the earth's surface inappreciable on a small globe.

(87.) It will now be easily perceived that the highest mountain on the earth, in comparison with the mass of the earth itself, is as nothing, and does not therefore affect the general spherical form of the earth. The very highest mountain is not quite five miles in height. A proportional representation of such a height on a globe six feet in diameter would be almost imperceptible. Its thickness may be estimated:—

Diameter of earth : height of Dwalaghiri = 7924 miles : 5 miles.

For a globe of six feet in diameter,

$$7924 : 5 = 6 : \frac{30}{7924} = {\cdot}00379 \text{ feet,}$$

or about $\frac{1}{26}$th of an inch.

CHAPTER VI.

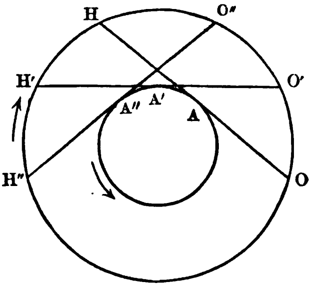
THE ROTATION OF THE EARTH ON ITS AXIS.

The earth (not the heavenly bodies) performs a daily revolution.

(88.) As, in regard to the form of the earth we have been compelled to correct the judgement of the senses, so must we, in regard to the *motions* of the heavenly bodies. Our eyes tell us (8) that there is a daily motion of them all from east to west, and a yearly motion of the sun among the stars in the opposite direction. The following paragraphs will show how the *apparent* daily motion of the heavens about the earth resolves itself into a *true* daily rotation of the earth on its own axis; and the *apparent* yearly motion of the sun, into a *true* yearly motion of the earth about the sun.

(89.) The daily motion previously described of the whole starry sphere in a circle around the pole from east to west, may be explained in two ways. Either the whole sphere of the heavens, with its stars moves from east to west in 24 hours about the earth which is at rest; or the earth moves in the same time from west to east.

Fig. 23.



In fig. 23, AA′A″ represents the earth; OO′O″ HH′H″ the celestial sphere; and HO the horizon of the place A. If the *earth* stood still and the whole heavens turned round from east to west in the direction H″H′H, at A the stars would become visible (or rise) when they had arrived at H in the east, and would remain visible as long as they remained in the part of their orbit above HO. When they arrived at O they would disappear (or set). In this way every point of the starry sphere would pass over any point in the fixed earth, from east to west, as it actually appears to do.

(90.) On the other hand if the *heavens* be supposed to stand still, and the earth to revolve from west to east, in the direction AA′A″, then will the horizon of the observer be continually changing. In the situation A his horizon is HO, and the stars at H are rising, while those at O are setting. When the rotation of the earth brings him from A to A′, then his horizon changes from the position HO to H′O′, and the stars which were rising as he was at A are now above his present horizon, and appear to have traversed the arc H′H. Those which were setting at A have during the same time traversed the arc O′O beneath his present horizon. In this way every point on the earth's surface must pass under the fixed heavenly sphere in the direction from west to east; and to an observer unconscious of his own motion along with the earth, the heavenly sphere will seem to be moving from east to west, as his movable horizon retires from the western stars and approaches the eastern. Both suppositions — 'the earth at rest and the revolution of the heavens from east to west;' or, 'the heavens at rest and the revolution of the earth from west to east' — are equally fitted to explain the daily motion of the starry sphere: the object of the following sections is to show that the last only is the true one, as indeed may almost be made sufficiently evident from the conclusions to which we should be driven by the adoption of the first supposition.

Probability of this rotation.

(91.) The innumerable stars of heaven, which seem to be fixed on the inner surface of the same hollow sphere, and therefore all at the same distance from us, are nevertheless, as astronomers show, at immensely different distances. A little consideration will suffice to convince us that it must be so. For example, the moon sometimes covers the face of the sun, sometimes it covers a planet or a fixed star; the planets cover fixed stars; Mercury and Venus on occasions pass over the face of the sun. It is an easy inference that the moon must therefore be nearer to us than the sun, and the planets than the fixed stars. Astronomers estimate the distance of the moon from the earth at 240,000 miles; that of the sun at 95 millions of miles; that of the most distant planet, Neptune, at 2,000 millions; and that of the nearest fixed star at not less than 19 billions. On the supposition that the celestial sphere revolves round the earth, it is plain that these bodies must in one day describe circles whose radii are the distances named. Expressing these radii by r, and using the formula $2r \, . \, \pi$ to express the whole daily orbits of these bodies; and, therefore, $\frac{2r \, . \, \pi}{24 \cdot 60 \cdot 60}$ to express the number of miles each moves in a second, we have for the motion of the moon in that time about 18 miles, for that of the sun about 70,000, of Neptune about 260,000, and of the nearest fixed star above 1,300 *millions* of miles. This last velocity exceeds all possibility of conception, even after we have realised that of light, which is about 192,000, or of electricity, about 300,000 miles per second.

If we attempt, on the contrary, to estimate the velocity which any point on the earth must have by its daily rotation, the results are not so overwhelming.

Taking the earth's circumference at 24,897 thousand miles, this gives a velocity of above 1,000 miles per hour for that of any point on the equator, or more exactly 1,467 feet per second, not quite one-third more than the known velocity of sound.

Improbability of the revolution of the celestial sphere.

(92.) If these comparisons tend to prove the rotation of the earth, by showing the unlikelihood of the contrary supposition, that supposition will appear still more unlikely when we reflect that by it, all these stars, whatever their differences in distance from the earth, are made to fulfil their extremely unequal paths in the same time, namely 24 hours. The magnitudes, too, of these bodies are unequal, as astronomers show; so that the sun, for example, which is more than 400 times the distance of the moon from us, and is 75 million times larger than it, must in its daily revolution move at more than 400 times the velocity of the smaller body. Besides, it is unlikely that the sun, which is a million and a half times the magnitude of the earth, should move round so small a body as the earth and be retained in its orbit by it; in fact, according to the law of gravitation, this is not merely unlikely, but impossible. To this let us add that the stars seem to move in circles parallel to one another, whose planes are perpendicular to the earth's axis. Their centres of motion, did the heavens really rotate, must therefore be along this merely imaginary line — the consequence is that the points of this line, *points without any attracting matter*, determine the daily motions of these enormous bodies, in contradiction to the law of gravitation. The influence of these points, too, must diminish as they are approached, for the stars nearest the pole move with the least velocity. All these considerations render the daily revolution of the celestial sphere highly improbable, and therefore increase the probability of the supposition that the earth has a daily rotation on its own axis.

(93.) The improbability, that the sphere of the heavens should revolve round the earth, being so great, we are justified in assuming the other of the two suppositions, which will account for the apparent daily revolution of the stars. It behoves us, therefore, to look for proofs of the earth's revolution on its axis.

(94.) If then, the earth revolve on its axis every 24 hours, there are certain results which may be stated theoretically as necessary consequences of this rotation, and which being found true by observation, may properly be considered as confirmations of the supposition.

Proofs stated.

First. The earth, as a consequence of its rotation, cannot be a perfect sphere, but must bulge towards the equator, and be flattened towards the poles; in fact must have the form of a spheroid. This flattening at the poles will show itself in

(*a.*) An increase of the length of a degree of latitude as we approach the poles; and

(*b.*) A gradual increase of the weight of bodies in proceeding from the equator towards the poles, arising from two causes connected with the earth's rotation:—

1. The stronger action of gravity towards the poles.
2. The action of centrifugal force at the equator.

The whole increase of weight should be in conformity with the united action of these two causes.

Second. If the earth rotate on its axis, bodies falling from great heights should have an eastward tendency.

Third. If the earth rotate, the plane in which a freely swinging pendulum vibrates should show changes corresponding to this rotation.

Fourth. The atmosphere moving freely on the surface of the earth should be subject to movements affected by the different velocities of different parts of the rotating earth's surface.

Experiment and observation show that these phenomena and changes do occur — all of them in such a way as that no other satisfactory cause can be found for them than the rotation of the earth. Their occurrence, therefore, may be taken as so many proofs that the earth does rotate on its axis. We proceed to consider each of them in detail.

First proof. The earth is a spheroid.

(95.) The earth is a spheroid. Physical science shows that by the motion of a body — more especially of a fluid or soft body, in a circle — a centrifugal force is generated, under which the particles of the body tend to fly outward, and the greater the velocity of rotation the stronger is this tendency. Geology leads us to believe that the earth originally was in a state of fluidity and that by the gradual cooling of its parts the solid surface has been formed. Undoubtedly the fluid earth, *if at rest*, would, like other fluids, have assumed a form exactly spherical, as is seen in drops of rain, and in molten lead when bullets are made. But if the earth rotate, the equatorial parts must move with greater velocity than the polar, and become subject to the greater action of centrifugal force. These parts must have been driven from the axis of rotation, and a consequent curving out at the equator and flattening at the pole must have occurred, till the form of equilibrium was attained. This form would be attained when the tendency which the particles at the equator had to fly off was so modified by the attraction upon them of the other particles of the earth, that the result was a pressure upon the equatorial particles perpendicular to the surface of the earth. The figure therefore of a section of the earth through the axis must be elliptical, and its equatorial greater than its polar diameter.

Consequent variation in length of a degree of latitude.

(*a.*) The change in shape here theoretically inferred must be seen in the various lengths of degrees of latitude. It is so. Indeed, so evident is it that it must be so, that Newton demonstrated the ellipticity of the earth's figure as a consequence of its rotation, and actually calculated the amount long before any measurements had suggested such a conclusion. His predictions, however, ultimately led to attempts at verification of the theory. The first were made in France, but on too small a scale, and with observations too imperfect to lead to any reliable result. About 1740, under the auspices of the Academy of Sciences, two arcs of meridians were measured, one near the equator in Peru, the other near the pole in Lapland. These measurements left no longer any doubt of the flattening of the earth towards the poles. About 1792-8, during the period of mental activity roused by the French revolution, two French astronomers, Delambre and Mechain, effected the measurement of an arc from Dunkirk to Barcelona. This has since been extended on the one side to the parallel of Greenwich, and on the other to the island of Formentera; its extremes

Country	Latitude of middle of the meridional arc measured	Extent of the arc	Length of the degree in English feet	Observers
Peru . . .	1° 31′ 0″	3° 7′ 3″	362,808	Condamine, Bougier.
India . . .	12 32 21	1 34 56	363,013	Lambton.
India . . .	16 8 22	15 57 40	363,044	Lambton, Everest.
Cape of Good Hope . .	33 18 30	1 13 17½	364,713	Lacaille.
United States	39 12 0	1 28 45	363,786	Mason, Dixon.
Rome . .	42 59 0	2 9 47	364,262	Boscovich.
France . .	44 51 2	12 22 13	364,535	Delambre, Mechain.
France . .	46 52 2	8 20 0	364,872	Lacaille, Cassini.
England .	52 35 45	3 57 13	364,971	Roy, Kater.
Russia . .	58 17 37	3 35 5	365,368	Struve.
Sweden . .	66 20 10	1 37 19	365,782	Svanberg.

include an angle of 12° 48′ 46·8″, the longest arc which has ever been measured. The measurement of this arc of itself gave sufficient proof of the spheroidal shape of the earth, by the increased length of one degree in the more northern parts of it. The evidence is still stronger when we compare the lengths of a degree as ascertained from measurements in different countries at widely varying distances from the equator. The table on the preceding page shows the results of such measurements.

The two diameters of the earth as deduced from these and similar measurements are

The equatorial .	41,843,330 feet=	7924·873 miles
The polar . . .	41,704,788 feet=	7898·634 miles
Their difference.	138,542 feet=	26·239 miles

$$\text{Flattening at the poles, } \frac{1}{301{\cdot}026}.$$

In the construction of artificial globes, the difference of 26·239 miles between the two diameters may safely be neglected. For a globe of 3 feet in diameter, the proportionate difference between the two would amount to about one-eighth of an inch, a quantity which it would be difficult to estimate with exactness. For all practical purposes the earth may rightly be considered as a sphere whose diameter is the arithmetical mean between the equatorial and polar diameters.

Consequent variation of weight from equator to the poles.

(*b.*) The flattening of the earth's surface should also be manifest in an increase of weight from the equator to the pole, and is so made manifest by experiment. The difference of weight or pressure of any body at the pole compared with that at the equator is as 195 to 194, the rate of increase being as the square of the sine of the latitude. It is important to understand clearly in what sense this statement is made. Manifestly, a body weighed by a balance at the equator, if taken to either pole, would only affect the balance as before, since both the body and the counterpoise would be equally affected by the removal. But if the *same* body presses upon a soft mass, for example upon clay, so as to make a deeper impression, the nearer the place is to either pole; or if the *same* body, being suspended in a high latitude, would break a stronger string than in a low; or, if it stretches the same spiral spring more, the nearer it is to the pole; these would be all evidences of a greater pressure downwards or weight, in the direction from the equator to the pole. In this sense the difference between the polar and equatorial weights of *one and the same* mass of matter is found to be $\frac{1}{194}$th part of the whole weight. How is this ascertained? By means of the oscillations of a pendulum. The duration of these oscillations depends evidently upon the intensity of the force which draws it downwards at any place. The stronger this force, that is the greater the weight of the ball, the quicker the pendulum will tend to fall and the shorter will be the vibration, all other things being equal. Now it is found that in a high latitude the *same* pendulum vibrates quicker than in a low one; near the pole, therefore, the pressure downward or weight of bodies is greater than near the equator. The number of oscillations in each case can be counted, and the intensities of the downward forces acting upon the pendulum be compared. The science of mechanics proves that the intensities of the forces thus acting are to each other as the squares of the number of vibrations made. For example, a pendulum of a certain length at the equator makes 86,400 vibrations in a day; but, when carried to the latitude of London, the same pendulum makes 86,535 vibrations in the same time. Hence, the pressure down-

Methods of ascertaining the variations of weight.

By the vibrations of a pendulum.

wards at the equator compared to that at London is as the squares of these numbers, or as 10,000 to 10,031·5. That is, a mass of matter weighing 10,000 lbs. at the equator would weigh at London 10,031½ of *the same* lbs.

By the length of a pendulum.

It was found by Galileo that the duration of the vibrations of two pendulums at the same latitude are as the square roots of their lengths. Hence, we have another mode of making the experiment on weight at different latitudes. The length must be increased where the vibrations for the same length are quickest, in order to produce vibrations of the same slowness. Consequently, the pendulum, beating seconds, must be made longer as we proceed towards the poles. According to exact measurements, the length of the seconds' pendulum at the equator is 39·01156 inches; at London it is 39·1393 inches; and at Edinburgh 39·1555 inches.

The fall of a free body through space, during the first second of its fall, is —

At the equator . . .	16·0437 feet
At London	16·0954 „
At Edinburgh	16·1020 „
At the poles	16·1269 „

The force of gravity, therefore, at the equator being 1, that at the pole is 1·005185; or, as already stated, the difference is $\frac{1}{194}$th of the whole.

(96.) The length of the seconds' pendulum at any place may be found by the aid of another, whose exact length and number of vibrations in a given time have been ascertained by experiment.

Let T, t, be the times of vibration, L, l, the lengths of the pendulums, and Z, z, the number of oscillations in a given time;

then $T^2 : t^2 = L : l$, and $T^2 : t^2 = z^2 : Z^2$;
$\therefore L : l = z^2 : Z^2$, in which equation

L is the length of the known pendulum, Z its known number of oscillations in a given time (e. g. an hour), z that of the seconds' pendulum in the same time, namely 3,600. Hence we find the length of the seconds' pendulum, for

$$l = \frac{\mathrm{LZ}^2}{z^2}, \text{ and if an hour be the given time,}$$

$$l = \frac{\mathrm{LZ}^2}{(3600)^2}.$$

By allowing the known pendulum to vibrate a given length of time (e. g. an hour) at two different latitudes, and counting its oscillations Z and Z′, we have

$$l : l' = \frac{\mathrm{LZ}^2}{z^2} : \frac{\mathrm{LZ}'^2}{z^2} = \mathrm{Z}^2 : \mathrm{Z}'^2$$

that is, the lengths of the seconds' pendulum at two different latitudes are as the squares of the number of oscillations of a pendulum of any length at these two latitudes.

Example.—If a pendulum at the equator in one day beat 86,400 times, and at another place 86,600; and the length of the seconds' pendulum at the equator be 39·01156 inches, then for the length of the seconds' pendulum at the other place

$$86400^2 : 86600^2 = 39{\cdot}01156 : x = 39{\cdot}191013 \text{ inches.}$$

And the attraction of gravity (weight) at these two places will vary in the same proportions.

Two causes for this variation in weight.

(97.) Two causes produce this difference in the weight of bodies at the equator and the poles, each of them connected with the spheroidal shape of the earth, and therefore confirmatory of its rotation on its axis. These are (*a*) the fact that at the equator a body is farther from the centre of the earth than at the poles, and (*b*), the absence of all centrifugal force at the poles.

1. The poles nearer the centre than the equator is.

(*a*.) If the earth were a perfect sphere, the tendency of all bodies on its surface towards its centre would be everywhere alike, for the simple reason that the sphere is symmetrical. But being elliptical, a body at the equator is farther from the centre than one at either pole; and mathematical investigation has shown that, owing to the form of the earth alone, its attraction ought to increase the weight of a body in going from the equator to the pole by $\frac{1}{590}$th part.

2. The centrifugal force at the equator.

(*b.*) If the earth rotate, then during the rotation, the parts at the equator, as they move in a great circle, must move with a velocity greater than the parts on any other parallel circle. At the poles there can be no rotatory velocity at all. This diminution of velocity, from about 1,000 miles an hour at the equator to 0 at the poles, causes a corresponding diminution of centrifugal force. Now the centrifugal force, by tending to throw the particles off from the earth, counteracts and diminishes their weight. Consequently, at the equator, where the centrifugal force is at its greatest, the attraction of the earth is most counteracted; but at the poles not at all. By this cause acting alone, the pressure of bodies acting at the equator is diminished $\frac{1}{289}$th.

Thus the diminution of weight at the equator due to the shape of the earth is $\frac{1}{590}$th; to the centrifugal force, $\frac{1}{289}$th.

And the sum of these two fractions gives the whole diminution of weight from the poles to the equator, as shown by the experiments with the pendulum; namely, $\frac{1}{590}+\frac{1}{289}=\frac{1}{194}$. (95.)

(98.) The following theoretical considerations are useful in the investigation of the centrifugal force at different points of the earth's surface, and consequent diminution of weight at these points.

(*a.*) As the motion about the earth's axis is uniform, and all parts of the earth describe their circle in the same time, 24 hours, the velocities (C, *c*) of two different points on its surface are to one another as the spaces they describe, that is, as the circumferences of their different parallel circles; hence, also, as the radii of these circles, and therefore (78) as the cosines of the different latitudes of the parts; that is, if B, *b*, express the latitudes of the two points on the earth, then—

$$C : c = \cos B : \cos b.$$

Or we may reason as follows:—Let R be the radius of the earth, and *r* the radius of the parallel circle belonging to the latitude *b*, then

$$r = R \,.\, \cos b \,\therefore\, 2\, r\, \pi = 2\, R\, \pi \,.\, \cos b\,;$$

and as the circumference of the earth at the equator (expressed by $2\, R\, \pi$) is in round numbers 24,000 miles, we have $2\, r\, \pi$, the circumference at the latitude $b = 24000 \cos b$. Let C equal the velocity per hour at the latitude *b*, then

$$C = \frac{24000 \cos b}{24} = 1000 \,.\, \cos b.$$

At the equator $b = 0$, and $\cos b = 1$, therefore the velocity per hour is 1,000 miles.

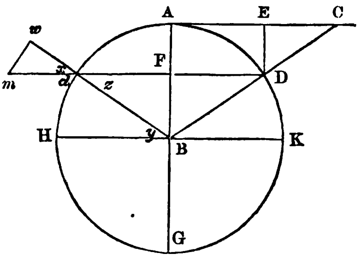
For latitude 60°:

$$C = 1000 \,.\, \cos b = 1000 \,.\, \cos 60^\circ = 1000 \,.\, \sin 30^\circ$$
$$= 1000 \times \tfrac{1}{2} = 500 \text{ miles per hour.}$$

That is, the parts of the earth in the latitude of 60° move with only half the velocity of the parts on the equator.

(*b*.) When a body moves in a circle, the direction of its motion at any point being that of the tangent, is perpendicular to the direction of the centripetal force, which is exerted along the radius of the circle. During the time in which a body at the point A of the circle ADK, fig. 24, moves to D, it would, in virtue of its acquired velocity alone, have proceeded to C, and thus have receded farther from the centre by the length DC. The extension of the radius BD to the tangent AC is thus the measure of the centrifugal force. It is the centripetal force which counteracts this tendency to fly off, and prevents the increase of distance from the centre. In circular motion,

Fig. 24.



therefore, the centripetal and centrifugal forces must be equal at every point of the circle, and the measure of the one will also be the measure of the other. Let DE be drawn perpendicular to AC, DF to AG, and let the arc AD be infinitely small, so that we may consider DC = DE = AF, and the chord AD (s) = arc AD (b): then in the semicircle ADG

$$\text{AF} : \text{AD} = \text{AD} : \text{AG}$$
$$\therefore \text{AF} : s = s : 2\,r$$
$$\therefore \text{AF} : b = b : 2\,r$$
$$\therefore \text{AF} = \frac{b^2}{2\,r}.$$

Now let v be the centrifugal force, and let the attractive force or weight of the body be taken as unity, so that the two forces are to each other as $v : 1$. In the time t, according to the laws of mechanics, the body by its weight would fall through the space gt^2; centrifugal force acting upon it would drive it therefore through a space equal to $v\,.\,gt^2$, which in the figure is AF.

$$\therefore v\,.\,gt^2 = \frac{b^2}{2\,r};$$

but as in the case of uniform circular motion, the velocity is equal to the space divided by the time, therefore (the velocity being c)

$$c = \frac{b}{t} \therefore b^2 = c^2\,t^2, \text{ and}$$
$$\therefore v\,.\,gt^2 = \frac{c^2\,t^2}{2\,r};$$
$$\therefore v = \frac{c^2}{2\,gr}.$$

For the centrifugal forces of motion in two different circles,

$$\text{V} : v = \frac{\text{C}^2}{\text{R}} : \frac{c^2}{r};$$

that is, centrifugal forces are to each other as the squares of the velocities divided by the radii of the circles in which the bodies revolve.

(c.) Let us apply this general law to the parallel circles of the earth. From (a)

$$\text{C}^2 : c^2 = \cos^2 \text{B} : \cos^2 b,$$

and $\text{R} : r = \cos \text{B} : \cos b$. Dividing:

$$\therefore \text{V} : v = \cos \text{B} : \cos b;$$

that is, the centrifugal forces at different parts of the earth are to each other as the cosines of their latitudes.

Now the cosines diminish as the arcs increase. Therefore, according to this result, the centrifugal force must diminish with the increase of latitude and the effects of attraction, or the weight, be less counteracted. But it is of importance to consider the directions in which the centrifugal force and weight work against each other on the earth's surface. Weight at any point on the earth acts always along the earth's radius from the point to the centre; but the centrifugal force at the same point works always from the centre of the parallel circle of the place, and therefore perpendicularly away from the axis of the earth. Hence it follows that these two forces are fully opposed to each other only at the equator, and that in all other places they form an angle. It is only at the equator, therefore, that the centrifugal force counteracts weight with all its influence; elsewhere the centrifugal force, already diminished, loses another part of its force, according to laws of mechanics, because it acts *obliquely* against weight. To determine the counteraction of weight by the centrifugal force according to the latitude of a place, let dm (fig. 24) represent the centrifugal force (v) for the place whose parallel circle has $d\,\text{D}$ for its diameter. Draw mw perpendicular upon the radius $\text{B}\,d$ produced, then, according to the parallelogram of forces, $wd = v'$, namely, that part of the centrifugal force which at d really counteracts the weight.

$$\text{Now } wd : md = \sin m : 1;$$

$$\text{and } \sin m = \cos x = \cos y = \cos \text{H}d\ (b);$$

$$\therefore wd : md = \cos b : 1;$$

that is, that part of the centrifugal force which actually counteracts the weight is to the whole centrifugal force as the cosine of the latitude is to unity; thus,

$$v' : v = \cos b : 1 \therefore v' = v \cdot \cos b.$$

For two places of different latitudes, we have

$$\text{V}' : v' = \text{V} \cdot \cos \text{B} : v \cdot \cos b.$$

$$= (\text{V} : v) \times (\cos \text{B} : \cos b)$$

$$=(\cos B : \cos b)' \times (\cos B : \cos b),$$
$$\therefore V' : v' = \cos^2 B : \cos^2 b; \text{ that is,}$$

the effects of the centrifugal forces against gravity are to each other as the squares of the cosines of the latitudes.

Hence, for a point at the equator and a point at latitude 60°, we have $V' : v' = 1 : \frac{1}{4}$, that is, although a point at 60° moves with half the velocity of a point at the equator, the diminution of weight by centrifugal force is only one fourth of that at the equator.

(*d.*) If now the centrifugal force at the equator be known, we can easily find its effect at all parts of the earth. Generally, if c be the velocity of rotation at any part (b), the centrifugal force $= \frac{c^2}{2\,gr}$.
At the equator c is nearly 1,500 feet per second; r the radius of the earth about 3,989 miles, or 3989×5280 feet; and g, by experiment, is nearly 16 feet. Hence,

$$\frac{c^2}{2\,gr} = \frac{(1500)^2}{2 \cdot 16 \cdot 3989 \cdot 5280} = \frac{1}{289} \text{ nearly;}$$

that is, at the equator the centrifugal force is $\frac{1}{289}$th part of the weight. If, therefore, we add to the space which a body falls at the equator during the first second of motion $\frac{1}{289}$th part of itself, we shall find the space through which it would fall if not counteracted by the centrifugal force.

(*e.*) With what increase of velocity should the earth revolve, so that at the equator centrifugal force should be equal to the force of gravitation, and thus bodies have no weight?

Let weight be taken as unity, and the centrifugal force or $v = \frac{c^2}{2\,gr}$. In the case supposed, $v =$ weight $= 1$;

$$\therefore 1 = \frac{c^2}{2\,gr};$$

hence $c = \sqrt{2\,gr} = 25{,}931$ feet per second.
This is a little more than 17 times the actual velocity at the equator; the earth must therefore revolve with about 17 times greater velocity in order that the bodies should have no weight.

Second proof of rotation. A stone from a height falls eastward.

(99.) Were the earth at rest, then a stone let fall perpendicularly from the top of a tower would reach the ground exactly at its foot. Or if the earth were merely a plane, moving from west to east, as the top and foot of the tower would then advance at the same velocity, the stone would still fall at the foot exactly. But if the earth be a sphere in rotation from west to east, the top of the tower will describe a greater circle in 24 hours than the foot, since it is necessarily farther from the centre of the sphere. The stone by its inertia will retain, during the time of its fall, the greater eastward velocity of the top; and therefore, while gravity is drawing it towards the earth, it will proceed eastward with a motion of its own. The result must be that it will deviate from the perpendicular line of the tower wall, and fall not at the foot, but to the eastward of the tower. That a stone does fall so from a great height has been confirmed by the most satisfactory experiments. Newton in 1679 declared that the course of a falling stone must tend eastward; Hooke and Guglielmini showed that it did so by experiments which were hardly of sufficient accuracy; but the experiments of Benzenberg in 1802 and 1803 were such as to set the fact beyond a doubt. Later still experiments were made by Reich in a deep well at Freyberg. The depth was 519·88 feet. The tendency eastward, which calculation from the law of gravitation and centrifugal force gives for that height, is ·090528 feet. The actual deviation found as a mean of numerous observations was ·092824. Considering the difficulty of exact observation under such circumstances, the difference between the results of theory and fact can easily be accounted for; it is less than $\frac{1}{40}$th of an inch.

Third proof of rotation: From apparent changes

(100.) If the earth rotate on its axis, the plane in which a freely swinging pendulum vibrates should show changes corresponding to this rotation.

of the plane in which a pendulum vibrates.

Careful observation of the movements of a freely swinging pendulum shows that such changes in its plane of oscillation do occur, and thus we obtain a proof of the earth's rotation.

The peculiarity of this proof consists in the fact of the rotation of the earth being made evident to the senses by the vibrations of the pendulum. It may, therefore, be called a direct proof. Experiments show that the point from which an oscillating pendulum hangs may be set in rapid rotatory motion, yet this does not cause the plane in which it oscillates to change. We find also, by experiment, that if the point from which an oscillating pendulum hangs be carried forward rapidly through space, this indeed causes a change in the *parts* of space through which the pendulum vibrates, but not in the *direction* of its vibration. E. g. — Suppose a heavy ball, hanging at the end of a fine thread, to be held suspended in the hand and set oscillating in any given direction, say from east to west. The ball may now be carried forward round the apartment, but still, if steadily carried, it will in all parts of the journey be found vibrating constantly from east to west.

Foucault's experiment: pendulum at the pole.

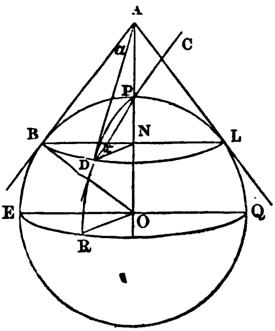
(101.) Similarly, though the earth rotates, and its parts are thus carried round in space, the direction in which a pendulum oscillates does not change; and from the phenomena consequent upon this we derive the proofs of the earth's rotation. Let us first take the case of a pendulum at the north or south pole, set to oscillate from a point of suspension upon the extended axis of the earth. Immediately under the plane of oscillation, and in the same direction, we shall suppose that a line is marked upon the surface of the earth along the horizontal plane. If the earth be at rest, then the pendulum will continue to vibrate immediately above this line, as there is no reason why the line and the plane of vibration should change their common direction.

But if the earth rotate from west to east, that is from right to left, still its rotation cannot affect the axis of the earth which always points to the same part of space. Neither does it affect the plane of vibration. The consequence must be that the line marked upon the earth's surface will turn round below the pendulum from right to left with the earth, and to a spectator who moves with the earth, but is unconscious of his motion, the plane of vibration will necessarily appear as if it were moving from left to right, that is from east to west. This, doubtless, is what might be observed at the pole, and of itself it would suffice to set the rotation of the earth beyond doubt. But, unfortunately, the difficulty of reaching the pole, renders it impossible to make the actual experiment.

At the equator.

(102.) Let us next take the case of a pendulum in vibration at the equator in any direction; for simplicity's sake let us say in the direction of the meridian northwards and southwards. A line marked upon the earth's surface on a horizontal plane immediately under the pendulum, would necessarily be a tangent to the earth at the equator, and being drawn from south to north would also be parallel to the earth's axis. If the earth be at rest, the pendulum would continue to oscillate above the marked line, for the same reason as in the former case. But if the earth rotates, then the line drawn will, by the rotation, describe a cylinder in space about the earth's axis as a centre, and, on account of the infinite distance of the celestial sphere, will point directly to the same part of the heavens as the axis, namely to the pole. The pendulum beating north and south, and never changing its direction, must therefore have the plane of vibration also constantly directed towards the pole; and thus the plane of vibration and line marked upon the earth will continue throughout the experiment to maintain the relation to each other which they had at the beginning: hence a turning of the plane of vibration does not occur at the equator; and thus, although the locality be easily reached, proof, by means of the pendulum, that the earth rotates, cannot be obtained there.

Fig. 25.



At a middle latitude.

(103.) We now come to consider the case of a pendulum vibrating in a middle latitude. Let O be the centre of the earth (fig. 25), P the pole, ERQ the equator, B the place of suspension, and draw BA, a tangent to the circle EBL, which is the meridian of the place B. Let a pendulum be set oscillating at B in the plane of the meridian; then BA will be the direction of a line marked upon the earth's surface immediately under the pendulum and in its plane of vibration.

If the earth be at rest, the plane of vibration and the marked line will remain unchanged in their relations as before. But if the earth rotates, then an apparent turning of the plane of vibration must take place, as at the pole, only it will not have described a complete circle during one revolution of the earth. Let the meridian PBE have changed place by the rotation of the earth, so that it has assumed the position PDR, a position such that the point D is indefinitely near to the point B. It is evident that, as the earth continues to revolve, the line AB will constantly be changing its direction, as at AB, AD, AL, which are all tangents to the revolving meridian of the place B, and form the lines which point northward at the horizon of B in the different positions of its meridian. In fact, the line AB will describe a cone in space while the earth is revolving. But, while the line pointing northwards thus changes its position, the plane of vibration in which AB at first lies remains unchanged, always moving parallel to itself. And thus, although the plane and the north line coincided at B, still, by even the smallest rotation of the earth, a change in their relation takes place; BA, as the plane of vibration, becomes DC, parallel to BA; and BA, as the north line, becomes DA; and the angle ADC=BAD is formed by the two lines. Hence, to the observer at B, unconscious of the earth's motion, the plane of vibration will appear to have moved to the right, while, in fact, it is BA (DA) which has actually moved to the left. The same thing will take place for each subsequent indefinitely small motion of the earth in rotation, and the plane of vibration will therefore continually appear to move more to the right or east. But it will not make a complete revolution in 24 hours, because the angle ADC or BAD is evidently less than BND or EOR,—the angles which the meridian describes at the earth's axis as it revolves. When BND=360°, which it does in 24 hours, BAD or ADC is less than 360°, that is, the movable line forming it (DC) has not yet completed a circle. The earth, therefore, at any middle latitude, must make more than one complete revolution before the line in which the pendulum vibrates can make one. The nearer the equator the place B is chosen, the greater is the time above 24 hours, in which the plane of vibration appears to describe a circle: for, as B approaches the equator, the angle BAD becomes always smaller in relation to the angle BND, and its increase in any given time by the revolution of the meridian bears a less proportion to the increase of BND.

(104.) It is easy to show that the exact amount of deviation, or the azimuthal motion of the plane of vibration, is proportional to the sine of the latitude.

The angle EOB, which is the latitude of B $= 90° - $ BOA; and because AB is a tangent BAO $= 90° - $ BOA. Therefore BAO $=$ EOB; that is, BAO is the latitude of B.

Let BND be x, and BAD be a, and BAO be b, then AB chord $a =$ BN chord x, and since a and x are indefinitely small,

$$\text{AB} \cdot a = \text{BN} \cdot x \;\therefore\; a = x \cdot \frac{\text{BN}}{\text{AB}} = x \cdot \sin b.$$

This will be true of every angle of rotation; so that if x be any angle through which the earth has revolved, and a the corresponding angle through which the plane of vibration has revolved, we shall always have $a = x \, . \sin b$. When the earth has made a complete revolution, the plane of the pendulum will have moved through 360° sin b.

At the equator $360° \cdot \sin b = 360° \cdot 0 = 0$.

At latitude 30°, $360° \cdot \sin b = 360° \cdot \frac{1}{2} = 180°$, or the plane of vibration describes a circle in 48 hours.

At latitude 52°, $360° \cdot \sin b = 360° \cdot 0{\cdot}788 = 284°$, or a circle is described in about 30 hours.

At latitude 60°, $360° \cdot \sin b = 360° \cdot 0{\cdot}866 = 312°$, or a circle is described in about $27\frac{2}{3}$ hours.

At the poles, $360° \cdot \sin b = 360° \cdot 1 = 360°$, or a circle, as before shown, is described in 24 hours.

(105.) This proof of the rotation of the earth was first suggested by Foucault, who also tested it by experiment, 1851. In such experiments practical difficulties occur in the proper suspension of the pendulum, and also in the resistance to continuous vibration offered by the air. As the meridian revolves, and the horizon changes, all things which it bounds change with it. Landscapes, fields, woods, houses direct themselves to different parts of space. Only the pendulum remains unchanged in its original direction. It thus becomes an instrument by which we may, as it were, *see* that the earth rotates.

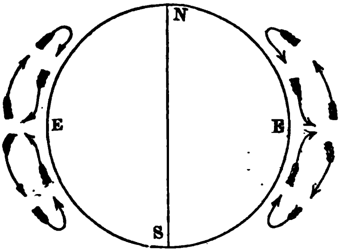
Fourth proof of rotation. The trade winds.

(106.) The atmosphere also affords proof of the earth's rotation. The cause of the trade winds, and those sometimes called the anti-trades, is perfectly explained by taking into consideration the unequal temperature of the air in different regions, and the rotation of the earth. Between the tropics the air is more heated than in the northern and southern regions. It becomes, therefore, specifically lighter and ascends; while the colder and denser air from the north and south hemispheres presses in to fill up the space occupied by the rarefied air which is left. Hence two streams of air occur in each hemisphere; in the higher regions of the atmosphere a stream of warm air from the equator towards the pole, and in the lower regions a stream of cold air in the opposite direction. If the earth did not rotate, then, at localities between the tropics and near the ground, direct north winds would perpetually blow on the north side of the equator, and direct south winds on the south. But because it rotates, and the whole atmosphere partakes of its motion from west to east, modifications arise. It is evident that the atmosphere must naturally have at each locality the rotatory velocity which belongs to the parallel circle at that locality. The streams of air from the poles have, therefore, a less velocity than that of the latitudes at which they successively arrive, and must necessarily remain behind in the rotation from west to east. They become, as it were, streams of air from the east, and move in a direction between their original one from the north and south poles, and that newly acquired. The stream in the northern hemisphere becomes a north-east wind (the well-known trade wind), being the modified result of the two motions; viz. that from the north pole and the acquired tendency from the east. So, the stream in the southern hemisphere manifests itself as a south-east wind, caused by the stream from the south pole and that from the east originated by the earth's rotation.

The anti-trade winds.

(107.) The heated air in the upper regions of the atmosphere flows on, as has been said, towards the North and South Poles. If the earth did not rotate, then there would have been perpetual south winds in those regions in the northern hemisphere, and north winds in the southern. But by the rotation of the earth, these upper currents carry with them to high latitudes the rotatory velocity due to their equatorial place of birth. So that, as they proceed, they gain upon the surface of the earth in its diurnal revolution, and become *relatively* winds with a motion from the west. Hence, in the northern hemisphere, the upper current becomes a wind from the south-west, as the resultant of its two motions. When at length this current returns to the surface, on being cooled and condensed as it approaches the North Pole, it acts as a powerful south-west anti-trade wind, blowing against the western coasts of the countries in middle and northern latitudes. The equatorial upper current, advancing towards the south pole, becomes similarly a north-west anti-trade wind. The curved arrows in fig. 26 represent the equatorial and polar currents of the air, the directions of which are changed by the revolution of the earth ENES on its axis NS.

Fig. 26.



Fifth proof. Analogy.

(108.) As the spherical form of the other planets affords an argument for that of the earth, so too the rotation of the other planets leads to the inference that the earth rotates. Astronomers not only observe the motion of the spots already mentioned, but also an evident depression at the poles of the planets. This depression is in proportion to the velocity of their rotation. Thus the planet Jupiter, which is 1,300 times the size of the earth, and which completes its revolution in somewhat less than 10 hours, has a flattening at each pole of about $\frac{1}{14}$th part of its radius. Mercury rotates on its axis in somewhat more than 24 hours, Venus in about 23½, Mars in 24½, Saturn in 10½, Uranus in 9½. The flattening at the poles of Mars is $\frac{1}{60}$ of its radius; of Saturn $\frac{1}{11}$.

The analogy of the planets may lead us here to anticipate the truth respecting the earth which is to be proved in our next chapter. They all move round the sun in elliptic orbits; and the axis on which they rotate is inclined to the plane of their orbit. The axis of Venus is inclined 75°; that of Mars 61½°.

The orbit of Mercury is completed in 88 days, that of Venus in 224, that of Mars in 687, and that of Jupiter in 4332 days. Saturn describes his orbit in 29 years, 5 months, 14 days; Uranus, in 84 years; and Neptune, in 164 years and a half.

CHAPTER VII.

THE ANNUAL MOTION OF THE EARTH ROUND THE SUN.

The phenomena of the heavens may be explained on two suppositions:

a motion of the earth, or

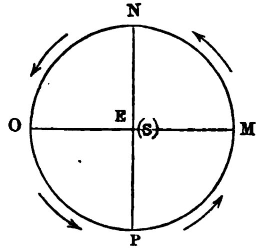
a motion of the heavenly bodies.

(109.) THE phenomena which lead to the inference that the sun has an annual motion round the earth from east to west have already been mentioned (10). But these phenomena may be equally well explained by each of two suppositions, namely, either, as above, that the sun moves round the earth at rest; or, that the earth moves round the sun at rest. In fig. 27, let E represent the earth in the centre, and let the sun be supposed to move in the circumference of the circle MNOP in the direction of the arrows. The observer at E will see the sun at M; in a quarter of a year after he will see it at N; in half a year at O; after three quarters at P; and again, when a whole year has elapsed, at M. Let us suppose, however, that the sun is at rest in the centre (S), and that the earth moves in the circumference in the same direction as that in which we previously supposed the sun to move. The observer at O will now see the sun as if situated at M; in a quarter of a year as he him-

at P, he will see the sun at N; half a year after

in M, the sun will appear at O, &c. On both

Fig. 27.



suppositions the observer will see the sun at the same seasons occupying the same positions in the heavens. As far as mere appearances are concerned, it is therefore indifferent whether the sun move round the fixed earth in the arc MN from M to N, or the earth move round the fixed sun in the arc OP from O to P. In the first case, if the sun be seen among the stars of the constellation Aries, it will really be there; in the second, however, the sun will only appear to be there, because the earth is moving among the stars of the opposite constellation Libra. In the first case, the sun would be seen among the stars of the constellation Taurus, because he would be really passing among them at the time; in the second, only because the earth is passing through the stars of the opposite constellation Scorpio, &c., &c.

The last the true one.

(110.) The mere probabilities of the case are in favour of the second supposition, that the earth moves round the sun. The sun is 1,384,472 times larger than the earth. That the smaller body, the earth, should move round the larger body, the sun, is therefore much more probable, than that the reverse should take place. If the sun revolve round the earth, its immense mass must sweep through space at the rate of about 1,200 miles in every minute. This may well strike us as improbable; but the laws of mechanical science prove also that it is impossible that a body so large should revolve about one infinitely small in comparison with it.

Probability of the earth's motion round the sun.

Proofs.

(111.) But direct proofs of the motion of the earth round the sun are to be found partly from astronomical observation of the planets, partly from that of the fixed stars. It can be shown:—

First: That all the phenomena of the planets, such as their movements, phases, eclipses, &c., can be perfectly explained on the supposition of the annual movement of the earth round the sun, and of a similar movement of the planets themselves.

Second: That certain apparent changes of the positions of the fixed stars in the heavens, caused by the aberration of light, are direct consequences of the earth's annual motion.

First proof of the earth's annual motion: From planetary changes.

(112.) The changes observed in the two planets, Mercury and Venus, may be first considered. These planets must be nearer the sun than the earth, for they not unfrequently come between the earth and it, passing as dark spots over the sun's disc. They are hence called inferior planets. The others, more distant than the earth is from the sun, are called superior planets. Mercury and Venus are never far from the sun in the heavens; the one never quite 48°, the other never quite 28½°. Venus is at the greater distance. At certain periods we see her after sunset in the west as the evening star, at others before sunrise as the morning star. Both of these planets undergo certain periodic changes, which may be observed better in Venus than in Mercury. These changes are three: namely, in their apparent distance from the sun; in their apparent magnitude; and in their phases. Let us suppose that we commence a somewhat lengthened series of observations upon Venus, on an evening when she has begun to appear shortly after sunset in the western heavens. The following evening she will be seen to have her place among stars more to the east than on the night before; hence she lingers a little later behind the sun above the horizon. This is repeated evening after evening, till at last she reaches her greatest distance eastward of the sun in somewhat more than six months from her appearance as the evening star. Then, after remaining for a time apparently immovable with respect to the sun, she begins to move

Changes of Venus.

In position.

towards it, approaching it night after night, and therefore moving in a westerly direction, setting each evening in shorter time after sunset, till at last she sets with the sun. This she does about three months after the time of her greatest eastern distance, or elongation, as it is called. Shortly after she has thus disappeared as the evening star, she is again seen as the morning star, rising in the eastern heavens just before the sun rises; and each succeeding morning preceding the sun by a somewhat longer interval. At this period her progress is necessarily towards the west of the sun, till at last she reaches her greatest distance (elongation) westward in about three months from the time of her appearance as the morning star. Another interval of rest occurs, she then begins again to approach the sun, becoming daily visible at a shorter period before sunrise, till at last she altogether disappears — that is, her eastward course has brought her once more to the sun. Soon again she appears as the evening star, and so repeats her former course.

(113.) These changes of position are all observable by the naked eye, but good telescopes show other changes which occur during the same period. When she first appears as the evening star it is as a full bright orb, but as she approaches her greatest eastern elongation, she gradually loses part of her circle of light, and when she has reached it, she shows only half a circle. During her subsequent approach to the sun she loses still more of her bright surface, and assumes a crescent form, with the horns turned towards the sun, becoming gradually smaller, till at last she disappears. On her reappearance as the morning star she has again the crescent form, but her horns are turned in the opposite direction, namely, from the sun. At the point of her greatest elongation westward she is again a half In form.

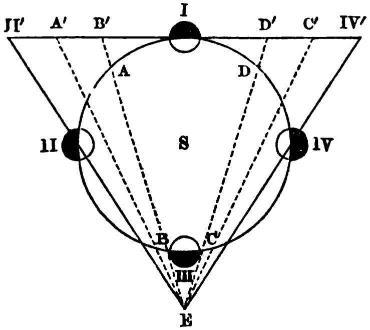
circle, and thence till her return to the sun she increases, at last becoming once more a fully enlightened circle.

n apparent magnitude.

(114.) Lastly, by means of the telescope, it is seen that from her first appearance as the evening star till her greatest eastern elongation and thence to her disappearance, she is constantly increasing in magnitude; and that from her first appearance as the morning star till her disappearance as such, she is constantly diminishing, so that when she again appears as the evening star she has the same apparent magnitude as at the first observation. These somewhat complicated changes in form, magnitude, and distance from the sun, are necessary consequences of a circular motion of Venus and the earth about the sun.

Explanation of these changes on the supposition of the earth's being at rest.

Fig. 28.



(115.) Let E, fig. 28, be the earth, supposed for simplicity at rest, and I, II, III, IV, different positions of Venus in her orbit round the sun, S. Although the sun and Venus are at different distances from us, we nevertheless consider them both as situated on the same surface, namely, the inner surface, so to speak, of the hollow celestial sphere, in the centre of which we suppose ourselves to be. Although Venus therefore is much nearer us when on one side of the sun as at III, than when on the other at I, she does not seem to us in describing her orbit to be moving in a circle from IV by I, II, III, and back to IV again, but rather to be moving backward and forward in a straight line extending directly from IV′ to II′. Let her move really in the quadrant from I to II, we at E fancy that she has moved from I to II′, or in the tangent parallel to the diameter from S to II. In the same way the actual motion from II to III appears as a motion from II′ to I, the true motion from III to IV becomes an apparent motion from I to IV′, and the true motion from IV to I an apparent motion from IV′ to I. In this way it may be easily seen how, without any change in her actual distance from the sun, or variation of her orbit round it, she seems to change the direction of her motion, and alternately to approach and depart from it.

(116.) On the supposition that she moves in an orbit nearer to the sun than that of the earth, we can also easily account for the changes in the form of Venus, called her phases. The sun always shines upon one half, and only one half, of the sphere of Venus. The enlightened half must of course be always that which is turned towards the sun. This half is sometimes turned full to the earth as well as to the sun, which is the case in the position I. Sometimes, however, this bright half is turned wholly away from the earth, as at III. We sometimes see only the half of the enlightened part, as at II and IV; sometimes more than the half of this part is turned towards us, as from I to II and from IV to I; and sometimes less, as from II to III and from III to IV. It is easy to perceive, therefore, that while she moves from I to II, the full bright orb must diminish to a bright semicircle, and that in moving from II to III the semicircle becomes ever less, diminishes to a thin crescent, and finally disappears. From III to IV the planet first shows herself as a thin crescent and then as a semicircle, which again from IV to I, gradually becomes a full shining circle. It is plain also that the direction of the horns in the part of the orbit from III to IV will be in the contrary direction from that to which they point, while Venus moves from II to III.

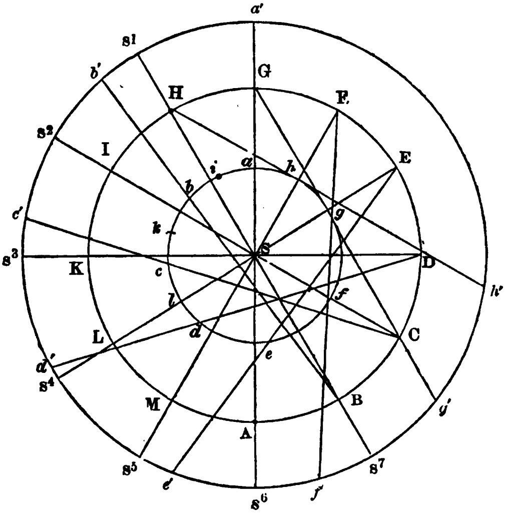
(117.) The changes in the apparent magnitude of Venus are easily accounted for, if we reflect that from I over II to III, while she is the evening star, she is continually approaching the earth, and therefore is seen from it under a constantly increasing visual angle, thus appearing to increase in magnitude; while during the period in which she is the morning star, namely, from III over IV to I, she is continually receding from us, and therefore apparently diminishing.

(118.) The simple explanation which is thus afforded of the changes of Mercury and Venus renders their motion round the sun evident enough. If we consider the earth to be also in motion round the sun (instead of at rest as before), then we shall find an equally simple explanation of the fact, that the time which Venus requires to reach her greatest elongation is so much longer than that required for her return to the sun again. The same explanation applies to Mercury, and thus the motions of Venus and Mercury become witnesses for the motion of the earth about the sun.

Explanation: on the supposition of the earth's being in motion round the sun.

(119.) First, it is easy to perceive that if Venus and the earth were to describe their respective paths in equal times, we should always see Venus at the same apparent distance from the sun — unchanged in magnitude and always in the same phase. For example, if Venus and the earth at the period of creation had been set in space on opposite sides of the sun, and appointed with equal times of revolution, or equal angular velocity in their respective circles, then we should always have seen her as a full bright orb. Had she been similarly set on the same side of the sun as our earth, in a direct line between us and the sun, we should never have seen her. The same supposition may be applied to any relation of the two orbits; under such circumstances these relations could never have been changed. But if Venus and the earth fulfil their paths round the sun in unequal times, then all these changes which we have already described, on the supposition of the earth at rest, will happen the same as before; but besides this, there will occur an inequality in the times which Venus requires to reach her greatest departure from the sun, and to return to it again. Venus completes her circuit in somewhat more than seven months; the earth in twelve. While, therefore, the earth makes one revolution, Venus makes one and almost two-thirds of a second. When Venus and the earth and sun are all in the same straight line, that is, when Venus is either in conjunction or opposition, she will be in the same part of the heavens with the sun, and appear either wholly dark or wholly bright; but when the straight

Fig. 29.



line from the earth to Venus is a tangent to the orbit of the latter, then she will have reached her greatest elongation. In fig. 27, let the smaller circle be the path of Venus, the middle one that of the earth, and the greater a section of the celestial sphere. The equal arcs *ab, bc, cd, de, ef, fg, gh, hi, ik, kl,* are the parts of her orbit which Venus traverses in different successive months; while the equal arcs AB, BC, CD, &c., are similar parts of the earth's path in the same times. If Venus be in *a*, and the earth in A, Venus is in opposition, the sun and Venus are in the same direction, and, after a short interval of invisibility, we see the latter as a full orb near *a′*. A month after this Venus is in *b*, and the earth in B; then we see the sun along BS in S^1, and Venus along B*b* in *b′* to the left, or eastward of the sun. After two months, with Venus in *c*, and the earth in C, we see Venus in *c′* still farther east from the sun in S^2. The arcs expressing the eastern elongation of Venus S^1b', S^2c', S^3d', S^4e', will thus continually increase till the maximum is reached, when the earth is in H and Venus in *h*, and the line H*h* is a tangent to the orbit of Venus at *h*. Venus does not, therefore, reach her greatest eastern elongation till a period of nearly seven months from her opposition. The arcs expressed in a similar way become henceforth less, and when Venus after three months, having passed over *i* and *k*, has reached *l*, the earth during the same period has passed over I and K to L. The angle between the sun and Venus as seen from the earth at L has again become nothing—the three bodies are in the same straight line—Venus is in conjunction. She has again reached the sun, now sets with him, and is, in fact, invisible for a short time. The two paths which Venus describes, first from the sun to her greatest eastern elongation, and then back again to the sun, are only apparently equal. To reach her greatest elongation she has traversed nearly seven of the parts into which her orbit is divided; while in order to reach the sun again she has traversed only three. Her departure requires seven months, her return but three.

(120.) The same explanation holds good in reference to the time during which Venus is the morning star. The lengths of the intervals in this case are reversed. She requires a longer time to return than to reach her greatest elongation.

Again, when the earth is in A and Venus in *a*, we see her full; when in H about half a circle; and when in L not at all. In the first case the bright side of Venus is wholly turned to the earth; in the second it is only half turned to it; and in the third it is wholly turned away.

Even so, by the motion of the earth from A over H to L Venus is ever brought nearer to the earth, and must therefore appear larger; while in the rest of her orbit she again recedes, and consequently appears less.

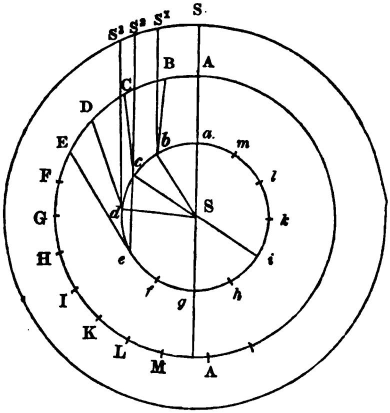
Changes of Mars in reference to the sun:

(121.) On the supposition of the earth's motion round the sun, we are again furnished with a simple explanation of the different appearances of the *superior* planets, namely, their apparent irregularities, their motions backward and forward in straight lines, and apparent periods of remaining fixed in the heavens. Mars is the nearest to the earth of the superior planets, and his apparent irregularities and changes of motion and magnitude are therefore most easily observed. If Mars rises when the sun sets, then he is in the south at midnight. If watched for some evenings after this, he seems to move in the heavens towards the western stars, that is, to go backwards from east to west. During the first period of making this movement his motion becomes gradually slower; after a time he seems to stand still altogether for a couple of days;

then he begins to take a directly opposite course, and advances among the stars to the east of his position, or forward from west to east, quickening his pace as he proceeds. After about a year the sun approaches him from the west and passes him, and for a few days he appears as a morning star, rising just before the sun. During this progress among the eastward stars the sun advances faster in the same direction than Mars; compared with the sun, therefore, Mars lags behind, but still advances slowly eastward, till at last the sun is again 180° from him, he is again in opposition, and rising when the sun is setting. After this, the same series of movements is repeated. His apparent magnitude diminishes from the period of opposition till that of conjunction; it increases till the period of opposition again. During the one period he must therefore have been withdrawing from the earth; during the other, approaching it.

And in reference to the fixed stars:

Fig. 30.



(122.) Let the inner circle in fig. 30 be the earth's path, the middle one that of Mars, and the outer the celestial sphere. The first is divided into twelve, the second into twenty-three equal parts, because the earth revolves in twelve, and Mars in about twenty-three months. Let the earth be in *a*, Mars in A, and as both proceed in their orbit in the same direction, namely, the earth from *a* to *b* to *c*, &c., and Mars from A to B to C, &c., then the arcs *ab* and AB, *bc* and BC, &c., will be traversed at the same times.

Explained by the earth's change of place in its orbit.

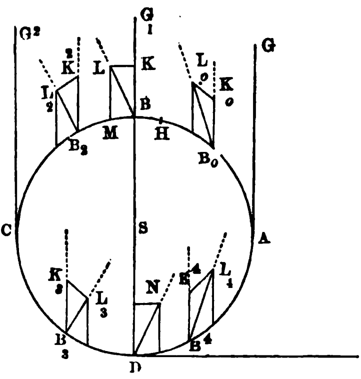
When Mars is at A and the earth at *a*, Mars is seen from the earth as if he were at S beside the fixed star S in the surface of the celestial sphere. A month after, the earth is at *b*. From this new position the same fixed star will now be seen at S^1, in a line bS^1, parallel to *a*S. But Mars, unlike the fixed star, being at a finite appreciable distance from the earth, and having reached a new position at B, is seen along *b*B to the right or west of the fixed star. He has, therefore, an apparent retrograde motion, since he appears to have gone westward from the fixed star. In the second month, the earth proceeds from *b* to *c*, Mars from B to C. We in *c* now see the fixed star in S^2, still in a line parallel to its first direction. Mars, too, is now seen in the line *c*C to the left or east of the fixed star. In the interval he appears to have arrested his retrograde motion, stood still a while, then moved slowly forward, with a direct motion from west to east. After three months, when Mars has gone over CD and the earth over *cd*, we in *d* see Mars at D still farther on the east of the same fixed star in S^3. This direct motion apparently increases, as when, at the end of the fourth month, the earth is at *e*, Mars is at E, and the fixed star is seen again at S^2 (since $cd = de$) in a direction ever parallel to itself; for in this position we attribute to Mars the whole of the motion by which he appears so much farther to the east of the fixed star. Thus, in a course of four months' observation, Mars apparently moves backwards, then stands still, and then moves forward again, at first slowly, afterwards with increased velocity; while in reality he has all the time been proceeding in a regular circuit round the sun.

The varying positions of Mars and the sun are explained by considering the angle under which they are seen from the earth in different parts of its orbit. If the line along which Mars is seen forms a straight line with that along which the sun is seen, then Mars and the sun are directly opposed to each other in the heavens, that is, in opposition, and Mars rises as the sun sets. The angle between them at the earth is of course 180°. As this angle diminishes to 90°, so does Mars approach the sun, and is, therefore, so much nearer his own setting at midnight. When the angle becomes 90° Mars is directly south at sunset, and sets at midnight. While the angle gradually diminishes from 90° to 0, Mars approaches nearer to the west at the hour of sunset, till at last (when it is 0) the sun and he set together. Mars is then in conjunction. When the lines along which they are visible begin again to form an angle, Mars will be seen on the other side of the sun before it rises, and gradually preceding it by a longer interval, will at last set as the sun rises and rise as it sets.

Second proof: from the aberration of light.

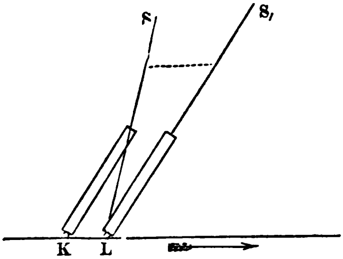
(123.) Bradley's discovery of the aberration of the light of the fixed stars affords an excellent proof that the earth is in motion round the sun. By the aberration of light is meant a slight change of place in the fixed stars during the earth's annual course; or, more correctly, a small apparent departure of a fixed star from the place which it would seem to occupy if the earth were at rest. This change of place is perfectly accounted for by taking the earth's annual motion into account with the time necessary for the transmission of light, and especially by considering the relation which the velocity of light has to the velocity of the earth in its path. In fig. 31, let S represent the sun, ABCD the earth's orbit, in the plane of which the star G is supposed to be situated. On account of the infinite distance of the star, it is seen from the different parts of the orbit in the parallel directions AG, BG^1, CG^2, DG^1; that is, these lines are the directions of rays of light from the star upon each of the points A, B, C, D. While the earth moves in the direction ABCD, at the point A it is evidently approaching the star, and at the point C receding from it, both times moving in the direction of the star's rays. At the points B and D, however, the direction of the earth's motion is at right angles to that of the light. The consequence must follow, that in A and C we see the star as if the earth were at rest in these points; but in B the star will be seen to the left of the position it occupied at A, and at D to the right of the same position. The explanation of this is derived from the principle of the parallelogram of forces. The velocity of light is 10,186 times greater than the velocity of the earth's motion in its orbit. Let the lines KB and HB represent the comparative velocities of light and of the earth's motion; KB will, therefore, be 10,186 times greater than HB. Let us now conceive a ray of light to be at K, when the eye of the observer is at H; then the ray and the eye will both reach the point B at the same time. The light as it enters the eye has, therefore, a compound motion; that of its own direction GB, and that of the eye from H towards B. Hence, according to the principle of

Fig. 31.



the parallelogram of forces, we see the star to the left of its proper place—namely, along the line BL in L—since BM = BH. In D, because the earth is moving in an opposite direction, the displacement is necessarily to the right of the true place of the star; and a telescope directed to the star G must be directed along DN, to the right of the true position of the star.

Fig. 32.



(124.) Suppose the star to be visible through a telescope at the instant when the earth is in the point L (fig. 32) of its orbit, and the star actually in the direction LS. The axis of the telescope, in order that the star may be seen through it under such circumstances, must be depressed, so that the light which entered it when the earth was at K may just have traversed the telescope while the earth was moving from K to L. Now the observer considers the direction of the telescope as that of the rays from the star, and therefore believes the star to be in the direction LS$_1$, while it is really in the direction LS. This error does not arise from the use of a telescope or of any other instrument; it is the same if the star be seen with the naked eye, the axis of which, in order that the star may be seen, must be directed with as much deviation from the true direction as the axis of the telescope: for, during the time that light takes to pass from the front of the eye to the retina, the retina has been moved onward by a certain distance, owing to the movement of the earth.

125.) It will be easily perceived from fig. 31, that in the point B only that the greatest change of place to the one side happens, and in the point D the greatest change to the other; and that the one displacement shows itself during the motion of the earth from A over B to C, increasing from A to B and diminishing from B to C; while the other occurs during the movement from C over D to A, increasing from C to D and diminishing from D to A. Generally, the aberration of light depresses the object on that side towards which the earth is moving at the time of observation. At the points B_2, B_3, B_4, B_0 of the orbit, the directions of the diagonals of the parallelograms show the directions in which the star is displaced. In these parallelograms, B_0K_0, B_2K_2, &c. represent the velocity of light, K_0L_0, K_2L_2, &c., the proportionate movement of the earth in its orbit. It is evident that the right ascension, declination, latitude, and longitude of a star are all affected by this source of error.

Which depresses a star from its true place as the earth moves towards it.

(126.) Light moves at the rate of 95,000,000 of miles in 8′ 13·2″, or about 192,000 miles per second. The earth moves in her orbit about 19 miles per second. The spaces moved over by each are, therefore, as 192000 : 19 = radius : tan. 20·25″. The size of the angle of displacement, therefore, is 20·25″. This displacement shows itself among the stars in the ecliptic as a yearly motion backward and forward in an arc of the celestial sphere equal to $2 \times 20{\cdot}25'' = 40{\cdot}5''$. At the pole of the ecliptic the displaced star describes a circle of diameter 40·5″, with the true place of the star for its centre. Among the stars between the ecliptic and its pole, the displacement occurs in ellipses, the greater axes of which are 40·5″, and the eccentricity of each ellipse is less the nearer the stars are to the pole of the ecliptic.

Angular amount of aberration.

The ecliptic is the earth's path, not the sun's.

(127.) Since the sun does not move round the earth, but the earth round the sun, there is, then, no actual path of the sun in the heavens, but a path of the earth. The plane, however, of the apparent path of the sun is exactly the same as that of the actual path of the earth; for the sun is always in the plane of the earth's path, and appears always in the point of the celestial sphere opposite to the earth's position. As the ecliptic and celestial equator intersect each other at an angle of 23½°, the axes of these two circles intersect with the same angle. But the ecliptic is the path of the earth, and the terrestrial equator lies in the plane of the celestial; therefore the terrestrial equator is inclined to the earth's path at 23½°, and the axis of the earth is not perpendicular to its path, but inclined from the perpendicular by the same angle; that is, it forms with the ecliptic the angles 23½° and 66½°.

CHAPTER VIII.

THE ELLIPTIC FORM OF THE ORBIT OF THE EARTH.

The earth does not move in a circle:

(128.) BUT does the earth really move in a circle round the sun? After a trial of the apparent magnitude of the sun, and the angular amount of its apparent motion in the heavens at different periods of the year, we answer, No. If the sun were always in the centre of the earth's path, and that path a circle, then we should be at the same distance from it in every part of the earth's orbit, since the radii of a circle are always equal: and we should, therefore, always see the sun with the same apparent diameter. Similarly, if the earth's orbit were a circle, its motion would then, according to mechanical principles, be uniform, and the sun's apparent change of place in the heavens should always be equal in equal periods of time. But observations show that the sun does not always appear of the same diameter, for its diameter, measured daily, increases at one part of the year, and diminishes at another; we must, therefore, be nearer it sometimes than we are at others. Its daily motion from west to east is also greater at one period than at another. From these observations it follows that the sun is not in the centre of the earth's path, and that the earth's motion in its orbit is not uniform.

As may be seen from its change of diameter.

The increase of diameter occurs at the same period as the increase of velocity, namely, from July 1 to January 1; and the decrease of both is also simultaneous, occurring from January to July. Hence it follows that we approach the sun at one part of the year with a velocity gradually increasing, and recede from it at another with a velocity gradually decreasing.

The following table gives the arcs apparently described in one day by the sun at different periods.

Date	Arc in one day	Date	Arc in one day
December . 31	1° 1′ 10·1″	July . . . 29	0° 57′ 23·3″
January . . 30	1 0 53·8	August . . 28	0 58 0·7
March . . 1	1 0 9·1	September . 27	0 58 56·9
„ . . 31	0 59 9·6	October . . 27	0 59 57·4
April . . . 30	0 58 11·5	November . 26	1 0 46·6
May . . . 30	0 57 29	December . 31	1 1 10·1
June . . . 29	0 57 11·8		

Thus the day on which the sun appears to move most in its orbit, is December 31, when he moves 1° 1′ 10″, and the day on which he appears to move least, is June 29, when his arc among the eastern stars is 57′ 11″.

(129.) The apparent diameter of the sun is, on

December 31 (winter solstice) . . 0° 32′ 35″
June 21 (summer solstice) . . . 0 31 31

The sun must, therefore, be nearer the earth in December than in June, in a proportion derived from these measurements:—

31′ 31″ : 32′ 35″ = 1891″ : 1955″ = 630 : 651, nearly.

That is, if we suppose the whole distance of the sun in December to be divided into 630 parts, its distance from the earth in June will be 651 of these parts, from which it is easily calculated that the sun's distance from the exact centre of the earth's orbit (his greatest and least diameters being at points diametrically opposite) is $\frac{1}{60}$ of the semi-diameter of the orbit.

But in an ellipse.

(130.) When a series of observations of the different apparent diameters of the sun, while the earth is in different points of its orbit, has been made, and the comparative distances of the earth from the sun at these different points ascertained, we find that they all lie in the circumference, not of a circle, but of a figure called an ellipse. One chief property of the ellipse is, that if from two given points within it, called the foci, there be drawn two lines to any point upon its circumference, the sum of these two lines will always be equal.

The true form of the earth's orbit was not known till the time of Kepler (1571-1630), who founded his deductions on the numerous and exact observations of his master Tycho Brahe (1546-1601). Kepler, choosing the planet Mars as the object of his investigation, was enabled at last to declare the three celebrated laws of planetary motion which bear his name.

Kepler's laws.

(131.) The two first of these relate to the form of the earth's orbit, and the rate of the earth's motion in that orbit. They are:—

First.

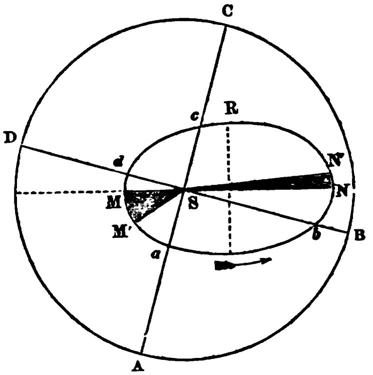
1. All the planets (and therefore the earth) move in elliptical orbits round the sun, which is situated in one of the foci.

Second.

2. The area of the portion of the orbit, which the line joining any planet (e.g. the earth) to the sun passes over in a given time, is proportional to the time.

(132.) The line supposed to join the planet to the sun is called the radius vector. In fig. 33, let M and N be the two extremities of the major axis of the ellipse which the earth describes about the sun, and S the focus in which the sun is situated. SM, SM′, SN, SN′, will be the radii vectores changing length according to the part of the orbit to which they are drawn. In M, the earth is at the point of its orbit nearest to the sun, called the perihelion; in N, at the point most distant, called the aphelion. The second law of Kepler shows the manner in which the velocity of the earth in its orbit varies according to the part of the orbit in which it is passing. If we take, for example, the arcs MM′, NN′ which the earth describes in equal times, the first from the perihelion M, the second from the aphelion N, the areas of the sectors MSM′, NSN′ are equal. It is evident from this that the arc MM′ must be greater than NN′, and consequently, the velocity of the earth's motion in its orbit is greater when in perihelion than when in aphelion; and, from analogous considerations, it is equally evident that the velocity of its motion diminishes from perihelion to aphelion, while from aphelion to perihelion it increases. The ellipse which the earth describes differs but little from a circle described with the greater axis for its diameter, on account of the small relative distance from the centre at which the sun is situated, namely, $\frac{1}{60}$th of the semi-diameter (129). It would be impossible for the eye to distinguish an ellipse, drawn in these proportions, from a circle. If in fig. 33 the larger circle represent the sphere of the heavens, then the ecliptic will be the intersection of the plane of the ellipse NRM*a* with this sphere. Let the points A, C, the extremities of the diameter AC, be the equinoctial points on the celestial sphere; then B, D, the extremities of a diameter perpendicular to AC, will be the solstitial points of the heavens. The major axis MN of the earth's orbit must be situated

Fig. 33.



in space in the same directions in relation to these points in which it is drawn in the figure, for we know that shortly after the time of the winter solstice, December 22, the earth is in its perihelion, January 1. That is, supposing the earth to move in the direction of the arrows in the figure, shortly after passing the point *d* it reaches the point M. It is evident from these two laws that when the sun appears to us of its greatest diameter, it is also apparently moving with the greatest velocity towards the eastern stars; and that when its diameter is least, its motion in this direction is slowest.

Bode's law of planetary distances.

(133.) The mean distances of the different planets from the sun are expressed by a series of numbers in which a remarkable relation exists. If we divide the distance of the nearest planet into four equal parts (each of which will be about $\frac{1}{10}$th of the earth's distance, or 9½ millions of miles), we get the distances of the other planets in a certain ascertainable number of such parts. The following table shows this; but it must not be forgotten that we have here only an approximation to these distances.

1.	Mercury	4	$= 4$
2.	Venus	$4 + 3$	$= 7$
3.	Earth	$4 + 2 \times 3$	$= 10$
4.	Mars	$4 + 4 \times 3$	$= 16$
5.	Ceres, &c.	$4 + 8 \times 3$	$= 28$
6.	Jupiter	$4 + 16 \times 3$	$= 52$
7.	Saturn	$4 + 32 \times 3$	$= 100$
8.	Uranus	$4 + 64 \times 3$	$= 196$
9.	Neptune	$4 + 128 \times 3$	$= 388$

The numbers in the last column thus express the mean distances of the planets from the sun, if the earth's distance be taken as 10. The distance of the $(n+2)$ planet in order from the sun will be expressed by $4+(2^n \times 3)$. This series is called Bode's Law of the Planetary Distances. Bode, a professor in Berlin, considering these relations, and observing a void in the planetary system as then known (1772), namely, between Mars and Jupiter, asserted that a new planet would probably be found in that space. This conjecture guided the observations of the astronomers who discovered the Asteroids. No account, *à priori*, or from theory, can be given of this singular progression; it is difficult, however, not to believe it something more than a mere coincidence.

Third law of Kepler.

(134.) Kepler's third law is:—'That the squares of the times in which two planets complete their orbits are to each other as the cubes of their mean distances.' This law enables us to compare the different planetary orbits. Having given the times of revolution of the different planets, and the distance of one of them, we can easily find the distance of the others. So, too, if we have the time of the revolution of one planet, and the distances of all, we can easily find the times of the rest. For example; the time of Jupiter's revolution once in its orbit is about twelve years, how many times is it farther from the sun than the earth, the orbit of which is fulfilled in one year? The cube of the earth's distance : cube of Jupiter's distance $= 1^2 : 12^2 = 1 : 144$, that is, the cube of the distance of Jupiter is 144 times that of the cube of the earth's distance: consequently the distance of Jupiter is $\sqrt[3]{144} = 5{\cdot}2$ times greater than that of the earth. So also, if we know that Jupiter is about 5·2 times farther from the sun than the earth is, we can find the time of Jupiter's revolution in its orbit. For $1^3 : 5{\cdot}2^3 = 1 : 140{\cdot}608$, that is, the cube of the distance of Jupiter exceeds that of the earth about 141 times; so many times according to Kepler's law does the square of the time of Jupiter's revolution exceed that of the revolution of the earth. Now $\sqrt{141} = 12$ years nearly.

(135.) With the knowledge of their distances, and of Kepler's third law, the times of the revolution of all the different planets, that is, the length of the year in each, may be found as above for Jupiter, by taking for the basis of our calculation the fact that the time of the earth's revolution is one year.

So, too, the mean velocities of the different planets may be calculated. Let their times of revolution be x years, and let the mean distance, considered as the radius of a circle in which each revolves, be r miles, then the general expression for their velocity of revolution per second is $\frac{2\,r\,\pi}{x \cdot 365 \cdot 24 \cdot 60 \cdot 60}$. E.g. at what rate per second do the earth and Jupiter move through space?

The earth's velocity =

$$\frac{2\,r\,\pi}{1 \cdot 365 \cdot 24 \cdot 60 \cdot 60} = \frac{r\,\pi}{1 \cdot 365 \cdot 12 \cdot 60 \cdot 60} =$$

$$\frac{95000000 \times 3{\cdot}1416}{1 \cdot 365 \cdot 12 \cdot 60 \cdot 60} = 19 \text{ miles per second.}$$

The radius of Jupiter's orbit by Bode's law is about 494 millions of miles, therefore Jupiter's velocity =

$$\frac{2\,r\,\pi}{12 \cdot 365 \cdot 24 \cdot 60 \cdot 60} = \frac{r\,\pi}{12 \cdot 365 \cdot 12 \cdot 60 \cdot 60} =$$

$$\frac{494000000 \times 3{\cdot}1416}{12 \cdot 365 \cdot 12 \cdot 60 \cdot 60} = 8 \text{ miles per second.}$$

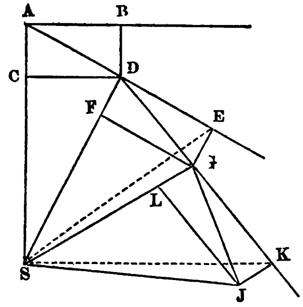
(136.) It was a suggestion of Kepler, that the sun holds the planets in their course by means of a central attracting force; it is the glory of Newton that he demonstrated the existence of this central force throughout the universe.

The laws of Kepler are but consequences of the universal law of attraction.

A body falling perpendicularly to the ground is drawn by the central attracting force of the earth.

A stone thrown from the hand, or a bullet shot from a gun, is under the influence of the propelling force from the hand or gun, and is also subject to the attracting force of the earth. The consequence is that it goes forward in a direction between the two forces, and ultimately reaches the earth, having moved along a curved line called a parabolic curve. Had the propelling force been strong enough the stone or bullet might have been sent so as, instead of falling to the earth, to have continued for ever, if in free space, revolving round the earth. The propelling force would send it straight forward, and the earth's attraction would deflect it continually. On a similar principle, the motions of the satellites round the planets, and of the planets round the sun, is explained. The attraction of the larger bodies for the smaller is the centripetal force; an original impulse bestowed at the period of creation is the propelling; the effect of the two is the motion of the revolving bodies in elliptic curves. The fact that these forces do exist is proved by the manner in which the planets revolve. We know that the planets move according to Kepler's laws, and it may be shown that Kepler's laws are necessary consequences of the action of two such forces. In reference to the second law, let S, fig. 34, be the sun, A, a planet, which under the propelling force alone would pass through the space AB in a given small portion of time; while in the same time the centripetal force of the sun would have drawn it from A to C in the

Fig. 34.



direction of S. The planet can only proceed in the diagonal of the parallelogram formed by the lines AB and AC, and will reach the point D at the end of this given time, say a second. According to the law of inertia, the planet would proceed during the next second in the direction and with the velocity of its previous motion, through DE = AD. If only acted upon by attraction it would during the same second have fallen through DF = AC. But, as in the former case, it can only move through DG, and therefore at the end of the next second it reaches the point G. Similarly at the end of the next again, it reaches not K nor L, but J, and so on.

If now we draw ES and KS.

$\triangle ADS = \triangle DES$ because $AD = DE$.
$\triangle DGS = \triangle DES$ „ EG is parallel to DF.
$\therefore \triangle ADS = \triangle DGS$. Similarly:
$\triangle GJS = \triangle DGS$.
$\therefore \triangle ADS = \triangle DGS = \triangle GJS$, &c.,

that is, the areas described in equal times are equal; or, are proportional to the times.

(137.) So, from the fact that all the planets move according to Kepler's third law, may be deduced the law of gravitation itself—namely, that attraction decreases as the square of the distance increases. By (98)

$S : s = \frac{C^2}{R} : \frac{c^2}{r}$, S, s being the spaces through which two planets move in a given time, C, c their velocities, and R, r the radii of their orbits. Let the time required for a planet in its revolution be t, then, generally, $c = \frac{2 r \pi}{t}$,

and by substitution $S : s = t^2 R : T^2 r$, or $\frac{S}{R} : \frac{s}{r} = t^2 : T^2$;

but as $r^3 : R^3 = t^2 : T^2$, by the third law of Kepler;

$$\therefore \frac{S}{R} : \frac{s}{r} = r^3 : R^3, \text{ or } SR^2 = sr^2.$$

Whence it follows that $S : s = r^2 : R^2$, or $S : s = \frac{1}{R^2} : \frac{1}{r^2}$, that is, the attraction of the sun upon the planets is inversely as the squares of their distances from the sun.

Summary. (138.) To sum up, the earth is about 7,924 miles in diameter. It turns daily on its axis from west to east with uniform velocity, so that a point on the equator moves about 1,500 feet per second. The sun is 1,384,472 times the size of the earth. Round this large body the earth moves in an elliptical orbit, and in the order of the signs of the zodiac. In one half of its orbit, namely, from aphelion to perihelion the velocity of the earth's motion constantly increases, from perihelion to aphelion that velocity diminishes.

The earth's path cuts the equator at an angle of 23½°. The positions of the axis of the earth, in all parts of its orbit, are always parallel to each other, and always at the same angle of inclination to the earth's path, namely, 66½°. The mean distance of the earth from the sun is 95 millions of miles. The whole orbit differs very little from a circle, the eccentricity, taking the semiaxis major as 1, is $\frac{1}{60}$. The whole orbit is nearly 600 millions of miles in length. This being gone round in a year, the motion of the earth in its orbit must be about 19 miles per second.

CHAPTER IX.

PHENOMENA RESULTING FROM THE FORM AND MOTIONS OF THE EARTH.

(139.) THE phenomena described in the previous pages have been arranged as *proofs* of the spherical form and the two principal motions of the earth, as well as of the elliptic form of its orbit. We shall now review these phenomena as the *consequences* of the things proved, and proceed to describe some other consequences resulting from them.

Consequences of the spherical form of the earth.

(140.) Evident consequences of the spherical form of the earth are :—

1. That we do not see the whole of distant objects.

2. That the horizon has always a circular form, the circle increasing or decreasing in diameter as our stand point is higher or lower.

3. That the earth is circumnavigable.

4. That stars, at first 1°, 2°, 3°, &c., distant from the zenith, appear *on* it if we travel towards them 69·16, 2 × 69·16, 3 × 69·16, &c., miles.

These phenomena have already been sufficiently described as proofs.

Consequences of the earth's rotation.

(141.) Consequences of the earth's revolution on its axis from west to east are :—

1. The apparent daily revolution of the whole celestial sphere in the contrary direction.

2. The rising and setting of the sun and moon.

3. The alternation of day and night (4).

4. The beginning and ending of the day earlier at places eastward than at places westward.

Revolution of the horizon.

The horizon of any place is an infinitely extended plane (33), which touches the earth's spherical surface at that place. Its position changes therefore with every change of place. In fig. 4, the lines FG, DE, HK may each be considered as a section of the horizon of the *same* place C, changing continually as the earth revolves from C over A to B. An observer while at C will see none of the stars below the line HK; when at A he will see none below DE; and at B none below FG. As the earth revolves, his horizon revolves with him: but he, unconscious of the change, considers the approach of the plane of the horizon to any star, as an approach of that star towards his horizon; and imagines that the star sinks behind it; while in reality, the horizon rises up and covers the star, as it were, from his sight. Although, therefore, we say, and cannot well help saying, that a star sets when it apparently goes down in the west, it would be more consonant to the actual truth of things to say, that it sets when it becomes hid by the revolving western branch of the horizon passing over it. Even so, the rising of a star is properly the passing of our eastern horizon under it. Thus, too, day as opposed to night is but the time during which the earth in its revolution is carrying our horizon under the sun; and night the interval between the passing of the western edge of the horizon over the sun, and the passing of the eastern under it. Or, again, the lines HK, DE, FG, fig. 4, may be considered as sections of the horizons of three different places, C, A farther east, and B still farther east. As the earth revolves from west to east, it is evident that the horizon of the place B must pass under the sun at G with its eastern edge some time before the horizon of A does so, and still longer before that of C passes under it. Thus the sun rises earlier to any place the farther east it lies. So the western edge F will pass over the sun at G before D, still longer before H, and sunset occur earlier to each locality in proportion as it lies eastward.

5. The trade and anti-trade winds are consequences of the earth's revolution on its axis (106).

Inclination to the horizon of the daily circles of the stars.

6. The greater or less inclination to the horizon of the daily circles of the fixed stars (25), is also a consequence of the revolution of the earth round its axis. These daily circles are all parallel to the celestial equator, since the stars appear to revolve round the axis of the celestial sphere, which is also the axis of the equatorial circle. Every place has its own horizon: but the equinoctial is the same circle for all places, and different in height above the horizon at different places. It and the horizon may, therefore, either coincide or intersect. If they intersect they will also bisect each other, both being great circles of the sphere; in which case one half of the equator will be above the horizon and the other half below. They may also intersect either at right angles to each other, or obliquely. These circles must, therefore, be considered in reference to each other under three modifications, (*a*) coinciding, (*b*) intersecting perpendicularly, and (*c*) intersecting obliquely.

Parallel position of the sphere.

(*a.*) If they coincide, the north pole will be the zenith, the south the nadir. But as the stars always move in their daily circles parallel to the equator, they also in this case move parallel to the horizon. This is called the parallel position of the celestial sphere. Only at the localities of the north and south poles can the equator and horizon coincide. The stars at these points never either rise or set, since they always move in circles parallel to the horizon. At the north pole only the stars of the northern hemisphere are visible; at the south pole those only of the southern; at each place the stars may be seen in *the whole* of their daily circle.

Right position of the sphere.

(*b.*) To the inhabitants of the earth at the equator, the celestial equator and the horizon intersect perpendicularly. The celestial equator there passes through the zenith and the nadir, while the horizon passes through the north and south poles. Circles parallel to the equator must there also be perpendicular to the horizon; and the sun and stars, which move in such circles, will rise on the one side of the horizon—ascend perpendicularly to their greatest height and descend perpendicularly to the opposite side, where they will set after a visible course of twelve hours duration. At the equator the stars are thus visible only in one half of their daily course. This is called the right position of the sphere.

Oblique position of the sphere.

(*c.*) But if the equinoctial intersect the horizon obliquely, the daily circles of the fixed stars, being parallel to the equator, will also be oblique to the horizon; and those of them which are nearest to the visible pole will fulfil the whole of their circle above the horizon, never setting. When their distance from the pole is greater than that of the horizon from it, they will begin to set in the lower part of their daily orbit; those just beyond this distance only dipping as it were beneath the horizon, those farther remaining longer, and performing a larger part of their circle, beneath the horizon; all, however, being more than half of their circle above it; till on the equinoctial the stars describe half of their orbit above and half below the horizon. The stars which move farther from the pole than the equinoctial is, perform less than the half of their daily circle above the horizon, and, indeed, so much the less the farther they are beyond the equinoctial, till at last their daily circle lies wholly below the horizon and they never seem to rise. Thus, among the fixed stars there are some which have the whole of their path above the horizon, some which have exactly the half, some which have more and some less than the half above, and lastly, some which have the whole of their path below it. Again, some are twelve hours above the horizon, some more, some less than twelve hours; and some are always above, while others are always below it. In such a position the celestial sphere is said to be oblique. This position is that which it has in Britain, and in all places situated between the equator and the poles. Different places differ in the degree of obliquity of the daily circles.

(142.) The dwellers at the poles see the stars of only one hemisphere in the whole of their daily path. The dwellers at the equator, on the contrary, see the stars of the whole celestial sphere, but only in half of their course. Those between the equator and the poles see the greater part of the starry heavens, some in the whole of their daily course, some in only a part of it. The stars rise and set only in the perpendicular and oblique spheres; in these only can we with propriety speak of the east and west points, or, indeed, of any points of the compass.

Consequences of the annual motion of the earth.

(143.) Among the principal consequences of the earth's annual motion to which we shall now attend, one is:—the apparent motion of the sun among the stars, in the order of the signs of the zodiac (109). The sun is always seen in that part of the heavens which is diametrically opposite to the actual place of the earth in its orbit (109). The ecliptic, or earth's path, is half above and half below the equator (38). Hence

1. The apparent motion of the sun among the stars.

it follows, that the earth is in the part of its orbit under the equator when we see the sun in its daily course above it, that is, from March 21 till September 23. These relative positions are reversed from September 23 till March 21.

2. The difference between a tropical and sidereal year.

(144.) The annual motion of the earth gives rise to the precession of the equinoxes; and so, to the distinction between a tropical and sidereal year.

The earth in its progress round the sun has fulfilled one revolution when it comes back to the point of its path from which it started. This can only be known by its being again seen in the same part of the heavens, that is, at the same point among the fixed stars. A year, therefore, is the interval between two successive occupations of this point. These intervals are always exactly equal, and the year so ascertained is called a sidereal year. It is somewhat more than 365 days 6 hours of mean solar time, namely, 365·25637 solar days. The time when the sun appears among the stars at the vernal equinoctial point, A, fig. 10, may be taken as the beginning of the sidereal year. That year will have elapsed when the sun, having passed successively over D_2, V, D_1, is again seen in the same point A among the stars. If the equator EQ and the ecliptic D_1D_2 always intersected in the same point among the stars, then would the time in which the sun revolved from the equinoctial point back to that point be always exactly equal to a sidereal year. But the equinoctial points A and V are constantly changing (41), and the celestial equator and ecliptic intersect each successive year at points V^1, V^2, ... A^1, A^2, each about $50{\cdot}1''$ westward from the last point of intersection; as if the equator were moving contrary to the course of the sun, or receding round the ecliptic from east to west by that distance. The equinoctial , therefore, comes against the sun in its apparent path, and the sun reaches it sooner than it should have done had the point kept its place unchanged. Hence from the equinox A to the same again at A′ is less than a sidereal year; less by the time which the sun requires to move through an arc of 50·1″, or about 20 minutes 24·34 seconds. The time of the sun's moving from the equinoctial point back to it again is called the tropical year; its meeting that point sooner than the former position of the point among the stars is called the precession of the equinoxes. The tropical years themselves are unequal to each other, on account of an unequal progress of the equinoctial points about the ecliptic; hence astronomers have recourse to a mean tropical year of 365 days, 5 hours, 48 minutes, 47·8091 seconds of mean solar time; that is, 365·24224 mean solar days.

3. The precession of the equinoxes.

This motion of the equinoctial points from east to west, and the consequent movement of the stars in reference to these points from west to east, is the reason that the twelve constellations of Aries, Taurus, Gemini, &c., must be distinguished from the twelve similarly named signs (41).

The cause of the precession of the equinoxes is:— that the sun attracts the bulging parts of the earth about the equator more than the other parts; for the amount of attraction is always as the mass of matter. When the sun is out of the plane of the equator, it tends to draw the equatorial parts of the earth towards its own place somewhere in the ecliptic. The equator is thus brought round before its time, as it were, towards the sun's place in the ecliptic.

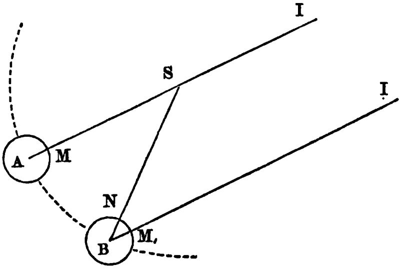
It is evident that if the earth were a perfect sphere there would be no precession; so, too, if the sun were always in the plane of the equator there would be none. Twice a year, therefore, at the equinoxes, when the sun is on the equator, the effect of precession is nothing; and, twice a year, at the solstices, when the sun is farthest from the equator, the effect of precession is at its greatest.

(145.) The difference in the length of a solar and a sidereal day is another consequence of the earth's annual motion round the sun.

4. The difference between a solar and sidereal day.

By a sidereal day is meant the time which the earth in its revolution requires to meet again the same fixed star on the same meridian. A solar day is the interval between the sun's passing any meridian and returning again to the same. The earth returns again to the same fixed star after exactly twenty-four hours, but not again to the sun till somewhat more than twenty-four hours have elapsed since the sun was last on the meridian. The reason is, that the earth has advanced in its orbit about 1° from west to east, so that it must turn round farther than one exact revolution in order to bring the sun again to the same meridian. Let the circle at A, a point in the earth's orbit (fig. 35), represent the earth; let B be another point of the orbit, S the sun, and I a fixed star. At A an observer with a telescope powerful enough to show him the stars by daylight, will see both the sun and the fixed star in his celestial meridian at the same time, along the line ASI.

Fig. 35.



If the earth be supposed to reach the point B in twenty-four hours from A, and to rotate in the direction of the arrows, then at B the star will be seen along the line BI parallel to its former direction, since it is infinitely distant, but the sun along the line BS.

It is thus manifest that the earth must turn farther round in the direction MN to bring the sun again on the meridian than to bring the fixed star upon it. Hence a solar day must be somewhat longer than a sidereal. This may also be understood by reference to the apparent motion of the sun, as follows:—The sun culminates to-day along with a certain fixed star. At the same time he is progressing in his annual course among the stars. To-morrow, when the same fixed star culminates, will be the end of a sidereal day; but at that time the sun will be among the stars eastward of the meridian, and must, therefore, culminate somewhat later than the fixed star. If the earth had no annual motion the sidereal and solar days would be of the same length.

Consequences of the elliptic orbit of the earth.

(146.) Among the evident consequences of the ellipticity of the earth's orbit are the changes in the apparent diameter of the sun at different periods of the year: and the changes in his actual distance from the earth between perihelion and aphelion.

Such, too, are the changes in the amount and velocity of the earth's motion in its orbit. The effect of these upon the length of the summer in the northern hemisphere, and, doubtless, on the whole climate of that hemisphere, is remarkable.

1. The longer summer of the northern hemisphere.

The sun remains nearly eight days longer each year north of the equator than he does south of it. In fig. 33, the four portions of the orbit *asb*, *bsc*, *csd*, *dsa* are evidently unequal on account of the curve being elliptical and the sun not in the centre; *a* and *c* being the equinoctial points and the earth moving in the direction of the arrows, *a* will be the vernal equinox. At *a* the sun begins to rise north of the equator, and remains above it in the northern hemisphere during his motion round to *c* by *b*NN′. But as he moves through equal areas in equal times, it is evident that in moving from *c* to *a* the time required must be less than from *a* to *c* again. Hence in the northern hemisphere, the seasons of spring and summer last 186^d 11^h 12^m, while autumn and winter last but 178^d 18^h 37^m.

2. One part of the inequalities of the solar days.

(147.) The inequalities in the length of the solar days are also partly caused by the ellipticity of the earth's orbit. This inequality, as far as it is the result of the elliptic form of the earth's orbit, may be understood by a reference to fig. 33. The velocity of the earth in its orbit is greatest in December and least in July. The apparent motion of the sun among the eastern stars must therefore vary, and with it the excess above a complete (sidereal) revolution necessary to bring the sun again upon the same meridian.

Consequences of the inclination of the axis of the earth.

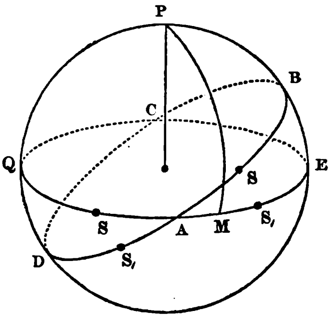
(148.) In order farther to explain the differences of the length of the solar day we will now consider the inclination of the earth's axis to the plane of its orbit.

1. Another part of the inequality of the solar days.

This inclination is an important cause of the differences in the length of the solar day. Were the axis not inclined to the plane of the orbit then the ecliptic and equinoctial would coincide: but, owing to this inclination, the two circles intersect at an angle of 23½°. By examining these, as drawn on a globe, it will be seen that a small arc of the ecliptic near the tropics is nearly parallel with the equinoctial, while an equal arc of the ecliptic near the equinoctial makes an angle with it. The sun in moving over an arc of the ecliptic about the tropics must, therefore, make more progress *eastward* than in moving over an equal arc about the equinoctial points. The consequence is, another variation at different seasons in the excess above a complete revolution necessary to bring the sun back to the same meridian.

(149.) If we suppose the two causes of irregularity removed; the earth's orbit made circular so that the sun should move uniformly in it (146); and the equator and ecliptic made to coincide instead of being inclined at an angle, then it is plain that the solar days would all be equal to each other, for the earth would always have exactly the same excess above a sidereal rotation to make. The length of the days thus brought about by the sun apparently moving *uniformly* on the equator would be a mean of the varying solar days which actually exist, that is, each would be a mean solar day. To illustrate this, let an imaginary sun s_1 be supposed to move uniformly on the equator QAE, while the real sun s moves on the ecliptic DAB, fig. 36. The latter may also, for the sake of simplicity, be supposed to move uniformly. Let them start together from the vernal equinoctial point A. The two suns, as they fulfil their circle in a year, must move through equal arcs in the same time; they will, therefore, always be in such positions on their respective circles, that the arcs AS, AS_1 are equal. They will each be 90° from the point Aries (A) together, and must come on any meridian at the same time at the summer solstice B. So they will pass the autumnal equinox C together, and at the winter solstice D will be again on the same meridian

Fig. 36.



together. Hence at these times the true noon and the hour of mean noon should coincide. A consideration of the figure will show that, as in equal times they describe equal arcs AS, AS$_1$, on proceeding from Aries, the imaginary sun S$_1$ gets more *eastward* than the real sun S. Now as the earth rotates in the same direction from A towards E, the celestial sphere will appear to rotate in the opposite direction from E towards A; the true sun S must therefore pass over any meridian, as PM, before the imaginary one S$_1$, or true noon will at that period precede mean noon; and generally the true time for that day will precede the mean. This precedence will diminish as the two suns approach B (the solstice), since the true sun is moving in a circle nearly parallel to the equator, and less in circumference (77); it, therefore, gains in eastward progress upon the imaginary sun till they coincide at B. After having passed the solstice for some time the imaginary sun will make less eastern progress than the real, seeing that it moves parallel to it, but in a larger circle; it will, therefore, in its daily revolution come first to any meridian PM, and mean noon will precede true noon, but gradually diminishing in precedence, till again they are together at the autumnal equinox. From that till the winter solstice true will precede mean time; they will again be gradually brought to coincide at the solstice, and after that, mean will begin to precede true time, then gradually diminish in precedence till they again coincide at the vernal equinox. The exact times of coincidence here mentioned are not strictly true, because of the first-mentioned cause of difference, namely, that the real sun does not move uniformly in the ecliptic.

Equation of time.

(150.) The interval by which the one of these times precedes the other is called the equation of time. It is evidently the same as the interval between time as shown by a sun-dial and by a well-regulated clock, since the dial shows true solar time, but clocks, watches, &c., mean time.

The mean and true time, by the two causes of their difference working together, do actually coincide about April 16, June 16, September 1, and December 25.

(151.) The imaginary sun, as has been observed, marks the mean solar day; a day which does not exist in nature, but yet is most important as that by which all the transactions of society are regulated. Being supposed to move uniformly round the equator, 360° in a year, it must, therefore, move about 59′ 8·3″ daily. The earth, in order to bring this sun upon the meridian at successive uniform periods, would have constantly to revolve 59′ 8·3″ above a sidereal revolution. The mean solar day is therefore always equal to a sidereal day, plus the time necessary to move over 59′ 8·3″, and may be defined as the time between two successive culminations of an imaginary sun moving uniformly on the equator.

Sidereal days in a tropical year, 366¼, solar, 365¼.

(152.) In the course of a year a fixed star makes one revolution more than the sun round the earth. This is evident by referring to fig. 35. For the angle NBM which the meridian must describe before completing a solar day, after having completed a sidereal, is equal to the angle ASB, which the earth has described about the sun in the same time, namely, a solar day. And it is manifest that, as the arc AB increases, so will the angle ASB or NBM. When the earth has completed its orbit, or 360°, the angle which the meridian must describe will also be 360°. But as 360° is a complete revolution, it will take just another sidereal day to describe it, so that in a solar year the number of sidereal days is one more than the number of solar days. When, therefore, the sun has made $365\frac{1}{4}$ revolutions, the fixed stars will have made $366\frac{1}{4}$. The lengths of the respective days must be inversely as the number of revolutions in a year; so that

$$\text{A solar day} : \text{a sidereal day} = 366\tfrac{1}{4} : 365\tfrac{1}{4}.$$

If then the mean solar day be taken as twenty-four hours, the length of the sidereal day may be found by the proportion

$$366\tfrac{1}{4} : 365\tfrac{1}{4} = 24^h : 23^h\ 56^m\ 4{\cdot}091^s.$$

Explanation of the calendar.

Its object.

The Julian calendar.

(153.) The sidereal year and the tropical year each involve fractions of days (144). Neither of these, therefore, can conveniently constitute the year by which historical events are registered and human affairs regulated. It is obvious that such years must consist of an even number of days, otherwise we should have some days of which the beginning would belong to one year and the end to another. In order to avoid this a conventional year is adopted, called the *civil* year. It consists of 365 or 366 whole days. The great object in regulating the lengths of particular years is to secure that the days, in which the sun is in the same place of the heavens in successive years, should have the same names. The arrangement by which this is effected is called a calendar. The ordinary year is made to consist of 365 days; and because this year is finished before the earth has reached the same point of the heavens at which it began its course, having yet nearly six hours of that course to perform, a day is added to every fourth year, which is called a leap year. This day is given to the shortest of the twelve months, February. The arrangement here described was made under the auspices of Julius Cæsar, 44 B.C. It is, therefore, named the Julian Calendar. Had the earth always attained the same point in its revolution after a course of exactly 365 days 6 hours, this arrangement would have been perfect. If we suppose the earth to commence its course at the vernal equinoctial point on March 25, then, when the civil year of 365 days had elapsed, it would be at a distance of 6 hours' progress from that point. Next year it would be 12 hours distance from the equinoctial point; next again 18; but on the fourth year the additional day of 24 hours would give it the opportunity of attaining the equinoctial point exactly. But as the mean tropical year is less than 365 days 6 hours by about 11 minutes, it is obvious that, each leap year, the earth under the arrangements of the Julian Calendar overshot the mark by the space which it moved during 44 minutes. Hence the vernal equinox was passed before the four civil years were finished, and the time of the equinox happened earlier than by a perfect calendar it should have done. In 371 years after the time of Cæsar, the equinox, from this cause, occurred four days before March 25, namely on March 21. The council of Nice sitting at that time, A.D. 325, made certain arrangements for keeping Easter on the faith that the equinox would always happen on March 21. But, regardless of the decrees of councils, the earth kept its ordinary pace; the error of 11 minutes each year went on increasing till the year 1582. By that time the equinox occurred 10 days before March 21, namely, on the 11th. To correct this, Pope Gregory XIII. ordained that the morning after October 4, 1582, should be, not the 5th, but the 15th of October. That day, which should have been March 11 next year, when the sun was again in the equinoctial point, became consequently March 21, and thus a restoration to the same state of things as when the council of Nice sat, was effected. To prevent the error in future was also necessary. This error of 11 minutes

Its defect.

The Gregorian calendar.

in excess each year, or rather 44 minutes each leap year, amounts to three days in 390 years. Gregory enacted that three of the years in every four centuries which by the Julian calendar would have been leap years, should not be so. The last year of each century was to be exempted from taking the additional day, except that of each fourth century counting from the Christian era. Thus, 1500, 1600, 1700, 1800 would all have been leap years by the Julian calendar, but by the Gregorian only 1600 was so; and 1900 will not be a leap year, but 2000 will.

Its arrangements.

By this arrangement:—

1. All years, the number of which is divisible by 4, are leap years, except those completing the centuries.

2. No year completing a century is leap year, except those divisible by 400.

Three days are by this means taken out of every 400 years as arranged by Cæsar; but this is too little, since three days should have been taken out of every 390. The actual *time* of the equinox must, therefore, still precede the *date* of the equinox (March 21) by a small quantity, which, however, would amount to one day in 3,600 years. This may be remedied by ordaining that years divisible by 4000 should not be leap years.

Adoption by different nations.

(154.) The reformed calendar was adopted by different countries as follows:—

Denmark	1582
England and Ireland	1752
France	1582
Germany, by Catholics	1584
„ by Protestants	1700
Holland, and the greater part of the Low Countries	1582
Hungary	1587
Italy	1582
Portugal	1582

Scotland	1600
Spain	1582
Sweden	1753
Switzerland, by Catholics	1584
„ by Protestants	1700

In Russia and some eastern countries it is not yet adopted. The consequence is, that our dates now differ from theirs by 12 days. The 5th of April in Russia is the 17th with us; 25th April there is 7th May here. The first is called the old style (O.S.), the other the new style (N.S.)

By England.

(155.) There are some circumstances connected with the adoption of the new style in England worthy of notice. Prior to 1752 the civil year in England commenced, not on January 1, but on March 25; and in that year the difference between the date of the old and new style amounted to 11 days. Therefore, two changes were introduced :—

1. The year was henceforth to commence on January 1.
2. Eleven days were to be left out by counting September 3, 1752, as September 14.

It will thus be seen that 1751 consisted, in England, of only 282 days, and 1752 of only 354.

Historical confusions consequent on its adoption.

(156.) Some confusion in historical dates has arisen from these changes, owing to uncertainty which of the styles is used; or whether the year is reckoned from January 1, or March 25. The date of the execution of Charles I. by the O.S. and old mode of beginning the year, is January 30, 1648; by the O.S. and new mode of beginning the year, it is January 30, 1649: but by the New Style fully adopted, it is February 9, 1649.

All dates prior to 1752, between December 31 and March 25, are liable to a discrepancy of a year, according to the style used. Dates are therefore sometimes so written as to express both styles, e.g. 1688-9, 1750-1; or $168\frac{8}{9}$, $175\frac{0}{1}$.

To express the difference between O.S. and N.S. in reference to the 11 days intercalated, dates are often written with the old as the numerator, and the new as the denominator of a fraction, e.g. August $\frac{3}{13}$ 1640.

$$\frac{\text{Jan. 30}}{\text{Feb. 9}}\ 1648\text{-}9 \text{ or } \frac{\text{Jan. 30}}{\text{Feb. 9}}\ 164\tfrac{8}{9}\quad \frac{\text{July 21}}{\text{Aug. 1}}\ 1686.$$

Or sometimes thus:—

January 30-February 9, 1649. August 3-13, 1640.*

Recurrence of day and date.

(157.) A common year of 365 days contains 52 weeks and one day. Hence days of the same date in successive years do not occur on the same days of the week. The first of January 1861 being Tuesday the same date in 1862 occurs on Wednesday, and in 1863 on Thursday. If all years consisted of exactly 365 days, then at the end of seven years the same dates would occur on the same days of the week as they did at the commencement of that period. The leap years prevent this regularity, so that the recurrence takes place sometimes after five, sometimes after six years, according as during the interval there have been one or two leap years.

The Solar cycle.

(158.) A cycle of 28 (4 × 7) years, however, causes a recurrence of the agreement of days and dates, since it contains seven leap years. This is called the solar cycle. These cycles are considered to have begun nine years before the Christian era. Hence to find the place of the year 1863 in the cycle, we must find how many cycles, and what part of a cycle has elapsed; thus: $\frac{9+1863}{28} = 66$ with a remainder 24; which shows that 1863 is the 24th year of the 67th cycle. The days of the week, with reference to religious festivals, are marked by the first seven letters of the alphabet. That letter which marks the Lord's day for any particular year is called the Dominical letter. When January 1 falls on a Sunday, as in 1860, A is the dominical letter, and the other days of the week are represented by B, C, D, E, F, G, in order. Had 1860 not been a leap year, January 1, 1861, would have fallen on a Monday, and Sunday would have fallen on the seventh day of the year, G the seventh letter would then have been the dominical letter. But as that year contained 52 weeks and two days, January 1 occurred on Tuesday, and the dominical letter was F. In fact, 1860 (like all leap years) had two dominical letters, A till February elapsed, and G for the rest of the year. It is evident that this cycle of twenty-eight years is modified by the Gregorian calendar, which makes the years divisible by 400 not leap years.

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* "'New Year's Day, the 25th of March;' this is the whole compass of the fact; with which a reader in those old books has, not without more difficulty than he expects, to familiarise himself. It has occasioned more misdatings and consequent confusions to modern editorial persons than any other as simple circumstance. In consequence of which we have Oxford carriers dying in January, or the first half of March, and to our great amazement going on to forward butter-boxes in the May following; and similar miracles not a few occurring."—*Carlyle*, '*Cromwell's Letters and Speeches*,' vol. i.

The Dominical letter.

The following somewhat complicated rule has been devised for finding the dominical letter. Divide the centuries by 4, and take twice what remains from 6; then add together this last remainder, the odd years above the even centuries, and the fourth part of these odd years, neglecting the remainder; divide the sum by 7, and the excess of 7 above the remainder is the number answering to the letter required.

By this rule the dominical letter for 1863 is D.

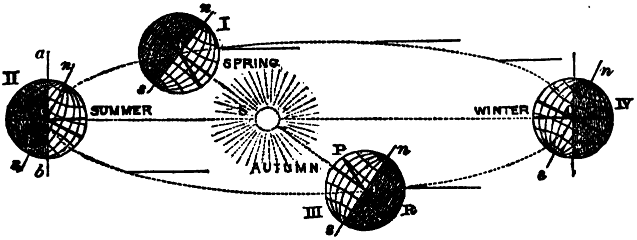
2. Changes in the declination of the sun.

(159.) The second consequence of the inclination of the earth's axis to its orbit is, that while the earth rotates, the sun appears to move in a daily circle constantly changing its declination. For, the sun being always in the ecliptic, and that being one half above and one half below the equinoctial, must appear to move through its daily arc sometimes higher, sometimes lower than the equinoctial, in the heavens. But the equinoctial is always the same height above the horizon of any one place; the sun, therefore, at that place apparently moves in an arc of the heavens of different altitude every day; at one time increasing its distance from the celestial equator northward, then returning, crossing, and departing from it southward, till it again returns. It thus passes over the same daily circle twice in each year, with the exception of its two tropical daily circles. On March 21 the equator is the sun's daily circle. From March 21 till June 21, the sun moves daily in the parallel circles between the equator and the northern tropical circle, every day approaching nearer to that circle. On June 22 it moves in it, and then begins to approach the equator. From that day till September 22, the daily circles of the sun again lie between the northern tropic and the equator. On September 23 the equator is again its daily circle. Thence till March 21, the daily circles lie between the equator and the southern tropic; on December 22, the southern tropic is the daily circle. There are, therefore, only two days in the year in which the sun rises exactly in the east and west points of the horizon; those in which the daily circle is the celestial equator, namely, March 21 and September 23.

3. Variations in the lengths of day and night.

(160.) The different lengths of day and night at different places, and their changing lengths at one and the same place, are also consequences of the earth's axis being inclined to its path. That half of the sphere of the earth which is turned towards the sun is always light, that turned away from it, dark. The boundary line between the light and dark sides is a great circle the plane of which is perpendicular to the line joining the centres of the earth and the sun. Every place in the enlightened hemisphere has day; in the other, night. The one hemisphere may be called the night side of the earth, the other the day side. As the earth is constantly revolving, turning some parts towards, and others from the sun, the night and day sides are continually changing. If the earth had neither a rotation on its axis, nor a forward motion round the sun, then the one side of it would have eternal day, the other eternal night. If again the earth had both motions, but the axis not inclined to the ecliptic, then all sides would have a constant succession of day and night, but twelve hours of day and twelve hours of night alternately, without variation. It is because the two motions take place with the axis inclined, that the variations which occur are caused. Sometimes, owing to this inclination, the sun shines more upon the north than the south hemisphere; sometimes the reverse takes place. Fig. 37 shows the earth in four parts of its

Fig. 37.



orbit round the sun. The axis *ns* is always inclined to to the plane of the ecliptic 66½°. This axis, too, during the earth's motion round the sun, remains, at any part of the orbit, always in a direction parallel to its direction at every other part. Hence, in different parts of the orbit, the parts of the earth's surface are differently exposed to the sun. In the part II the north pole is turned towards the sun, and the south from it. In IV these positions are reversed. In I and III neither pole is turned away from, or towards the sun; in these positions, therefore, the boundary line between the night and day sides of the earth passes through both poles. In II, that boundary reaches $23\frac{1}{2}°$ behind the north pole, and only to a point equally far before the south; that is, the north polar circle is wholly light, and the south polar circle wholly dark; for $soa = 90°$ but $son = 66\frac{1}{2} \therefore noa = 23\frac{1}{2}°$. In IV the boundary line is reversed in its relation to the polar circles. In I and III the sun is perpendicular to the equator, that is, the line joining the centres of the sun and of the earth in falling on the surface of the earth strikes the equator. In II this line falls upon the northern tropic (Cancer). In IV it falls upon the southern (Capricorn). When the earth during its annual revolution is at I, on March 21, and at III on September 23, all places have equal length of day and night, namely, twelve hours of each, because all places during one revolution are as long on the day side as on the night side. It is evident from the figure that, in these positions, exactly half of each parallel circle is on the night side, and half on the day side; and as the earth revolves uniformly, the times during which a place, on *any* parallel, is on either of these sides must be equal. In II, on June 21, the northern hemisphere within the polar circle remains constantly on the day side, while the earth revolves, and the southern hemisphere within the polar circle is constantly on the night side. In IV, on December 22, these conditions are reversed.

(161.) An inspection of the figure will show:—

(*a*.) That on June 21, when the earth is at II, the days are longer in the north than in the south hemisphere, because of the inclination of the axis towards the ecliptic; and that the days are so much the longer in the north, and so much the shorter in the south, the nearer a place is to either pole; because as the earth rotates, places on any parallel circle are a longer or a shorter period in the day or the night half of the earth, as the parallel is nearer the equator or the pole.

(*b.*) That during the progress of the earth from I to II the boundary line between the night and day sides of the earth, which in I passes through both poles, changes to the circle whose edge is marked in II by part of the line *ab*, in which position the boundary line passes 23½° on each side of the poles. As this change proceeds, the day of any place on the northern hemisphere gradually lengthens, and that of any place on the southern gradually shortens. With the earth in II, the north hemisphere has its longest day, and the south its longest night.

(*c.*) That the north pole during this quarter of the year is always on the day side, and the south pole always on the night side; the first, therefore, has continuous day, the second continuous night.

As the earth proceeds from II to III, the boundary line changes so as again to pass through the poles, and day in the northern hemisphere during this quarter gradually becomes shorter, in the southern gradually longer; until day and night become equally long in both hemispheres when the earth reaches III. As the north pole from I to III has been constantly on the day side, and the south pole constantly on the night side, it is evident that, during this period of half of a year, the north pole has daylight and the south pole night. While the earth moves from III to IV, the phenomena occurring between I and II are reversed in relation to the different hemispheres and poles; from IV to I, the phenomena between II and III are reversed.

Explained by the sun's apparent motion in the ecliptic:

(162.) In explaining the varying length of the day by the sun's apparent motion in the ecliptic, we should say, that when the sun is in the equinoctial, it shines equally upon the northern and southern hemispheres. When north of the equinoctial, it shines more upon the surface of the northern than of the southern hemisphere; but when south, more on that of the southern than of the northern.

When the sun is in the vernal equinoctial point, March 21, the boundary line passes through the earth's axis and the poles, and, by the revolution of the earth about its axis, every part of the earth is an equal time on the night and day side; therefore, all the world has then equal day and night. When the sun passes north of the vernal equinox, the light boundary passes beyond the north pole and does not reach the south; and the more the sun's declination is north of the equinoctial, the more its light passes beyond the north, and falls short of the south pole. When at last, on June 21, the sun has reached the northern tropical circle, the light boundary falls 23½° beyond the north pole, and falls short of the south pole by 23½°. In a similar way the changes during the other quarters of the earth's orbit may be explained.

and by the sun's varying daily arcs.

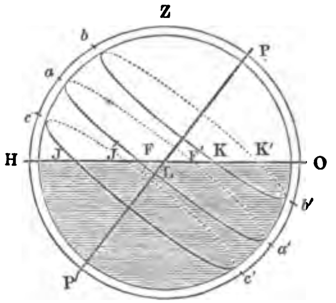
(163.) The variations of the lengths of the days may also be explained by reference to the daily arcs which the sun describes at different periods of the year.

As the declination of the sun changes, he daily rises higher above, or sinks farther below, the equator. His midday altitude at a given place will therefore increase or diminish daily, and there must be corresponding changes in the daily arc which he describes, or rather appears to describe, while the earth revolves. The effect of these changes upon the length of the day will vary, according as the earth's position, in reference to the sun, is that of the oblique, the parallel, or the right sphere. (141.)

The sun's arcs in the oblique sphere.

(164.) In the oblique sphere, let L (fig. 38) be the position of London in the centre of the celestial sphere, as an observer at that place naturally imagines it to be. Let the inner of the two circles represent a section of that sphere, which appears to turn round in the direction F*a*F′. Let the outer circle represent the meridian of London, which appears to the observer, unconscious of the earth's motion, to be fixed in a certain direction in the heavens, and abiding, while the celestial bodies pass it in their daily revolutions. HO will represent a section of the rational horizon of London, and P the pole will be 51½° above it, since the latitude of London is 51½°. While the celestial sphere seems to revolve around the axis PP′, the place of the equinoctial will be always on the same point *a* of the meridian PZH; and any other parallel will always meet the fixed meridian in the same point: thus, the Tropic of Cancer, 23½° north of the equinoctial, will always be in the point *b*, and the Tropic of Capricorn in *c*. On March 21, when the sun is in the equinoctial, and his noonday height at *a*, his diurnal circle will coincide with the equinoctial; he rises in F, passes over *a*, and sets in F′. His daily circle is evidently cut into two equal parts by the horizon, and therefore the day is equal to the night. The declination increases north of the equinoctial, from that time. On June 22, the greatest declination and noonday height northwards is attained. His

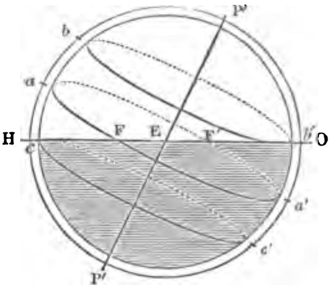
Fig. 38.



daily arc now coincides with the Tropic of Cancer. He rises at K, passes over *b*, and sets at K′. It is evident that the greater part of this circle is above the horizon, and that the day is longer than the night at London. In fact, on June 22, the day is about 16½ hours in length, and the night 7½. This is the longest day and the shortest night; for, from this point the declination and midday height again become less. The sun passes over all possible parallels in his daily arc, again reaches the equinoctial, and again describes a daily circle coinciding with it, half above and half below the horizon. It is now the autumnal equinox. The declination (0 at the equinox) increases from this point south, the sun similarly describing daily circles, corresponding to all parallels between the equinoctial and the south tropic at *c*, 23½° south from the equinoctial. This tropic and his daily circle coincide on December 22. He now rises at J, passes over *c*, and sets at J′. Here he is evidently much less above the horizon of London than below it. The length of the day and night is now reversed, compared with the same at the northern tropic. Night is now 16½ hours, and day 7½. It is the time of the shortest day and longest night. The sun now begins to approach the equinoctial — each day describing a somewhat longer daily arc, always, however, less than a semicircle, and therefore day always shorter than night — till at last it is again on the equinoctial, and day and night are once more equal. From this point it proceeds as before.

(165.) The rising and setting of the sun, as above described for London, is similar for Edinburgh, Paris, Madrid, and all other places in the Northern Hemisphere, where the point *c*, or the Tropic of Capricorn, is above the horizon. Now, this Tropic is 23½° from the equinoctial, and the height of the equinoctial above the horizon of any place is the complement of the Polar height. (59.) At London, the complement of the Polar height is 90°−51½=38½. The point *a*, therefore, being 38½° above the horizon, *c* also must be above it. But at a place whose latitude is such that the complement is less than 23½, *a* will be less than that number of degrees above the horizon; and therefore, the point *c*, and the whole tropical circle, must be below it. The changes of day and night will therefore occur differently at such a place, even though the sphere be still oblique. Let fig. 39 represent the celestial sphere, as seen by an observer at some place in latitude 66½°; so that *c* is exactly on, but never above, the horizon. When the sun is on the equator, his daily circle, F*a*F*a′*, is divided equally as before; and here, also, at the equinox, day and night are equal. When he progresses to his greatest northern declination at *b*, the daily arc *b′b* has no part below the horizon. The sun just touches the horizon at his lower culmination *b′*; then rises again as the earth continues to revolve. The longest day here lasts 24 hours, and the shortest night is 0. Immediately on the point *b* being reached, the sun begins to approach the equinoctial, and rises and sets as before; and the nights increase from 0 to 12 hours, when he is again on the equinoctial. When he reaches the southern tropic, and *c* is his midday height, no part of his daily arc is above the horizon; the night is then 24 hours long, and the day 0. Immediately, however, he begins to move towards the equinoctial, and once more sets and rises daily.

Fig. 39.



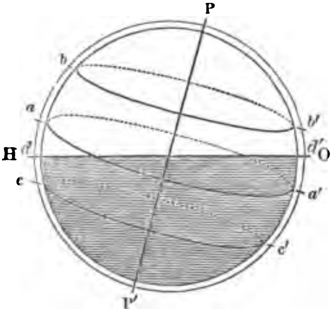
For the inhabitants at 66½° latitude there are thus:—

1. A period, from the winter to the summer solstice, during which the days increase from 0 to 24 hours in length.

2. A period, from the summer till the winter solstice, during which the nights increase from 0 to 24 hours in length.

(166.) If, however, the latitude of a place be more than 66½°—e.g., Nova Zembla, 75° N.—then the height of the equinoctial above the horizon is only 15°. Hence, *c* (fig. 40) must be 8½° below it, and *b′* 8½° above. The sun, in his progress towards the Tropic of Cancer, will evidently have reached a parallel, no part of which is below the horizon, when he is yet 8½° south of that tropic; and whatever be the number of days necessary for him to rise from *d′* to *b′*, and to return again to *d′*, so many days must he be without setting to the people of that latitude. Similarly, during his southern declination, he will never rise to them, during his progress from *d* to *c* and back again.

Fig. 40.



In latitudes greater than 66½°, the year is thus divided into: —

1. A period, commencing from some time before the vernal equinox, of rising and setting of the sun, with days becoming gradually longer and nights shorter, till the day is 24 hours long, and nights 0.

2. A period during which the sun does not set.

3. A period, commencing shortly before the autumnal equinox, during which the sun rises and sets, and the

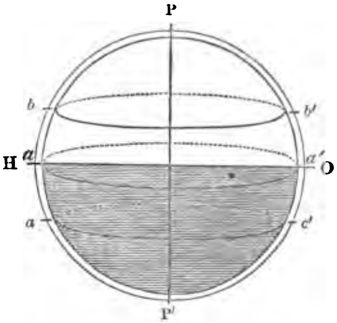
nights become gradually longer, and days shorter—the one to 24 hours, and the other to 0.

4. A period during which the sun does not rise.

The sun's daily arc in the parallel sphere.

(167.) In the parallel position of the sphere, the equator is the horizon, and the situation of a supposed observer is at the pole, with latitude 90°. As all the daily circles of the sun are parallel with the equinoctial, whatever be his position on the ecliptic, it is plain that he must be above the horizon, in this position, during the whole period of his rise, from the vernal equinox till the summer solstice, and back again till the autumnal equinox; that is, from *a* to *b*, or *a′* to *b′*, and back (fig. 41). There is thus continuous daylight of six months at the pole, during which the sun never sets, but wheels in circles parallel to, and above the horizon. And, similarly, all the sun's daily circles are below the horizon, while he changes his declination from *a* to *c*, or *a′* to *c′*, and back again. There is thus continuous night of six months at the pole.

Fig. 41.



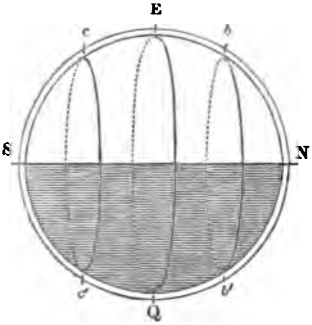
(168.) In the positions thus described for different latitudes, the Southern may be substituted for the Northern Hemisphere, and the same phenomena will be true of each, but at different periods of the year. What happens in the north at one period will happen in the south at the opposite period. When the North Pole has continuous daylight of six months, the South Pole has continuous night. When a place, 51½° north latitude, has long days and short nights, a place 51½°

South, has short days and long nights. So, when a place 66½° north has a day of 24 hours, a place 66½° South has a night of the same length.

The sun's daily arc in the right sphere.

(169.) In the right position of the sphere, the pole is on the horizon, and the place of a supposed observer is on the equator. Here all variation in the length of day and night disappears. Let SN (fig. 42) be the horizon, S being the South Pole, N the North, and EQ the celestial equator of an observer placed anywhere on the terrestrial equator. It is easy to see, from the figure, that whatever be the place of the sun on the ecliptic, his daily circle is cut into two equal parts by the horizon, and therefore, day and night are always equal.

Fig. 42.



Latitude	Longest day		Shortest day	
Degrees	Hours	Minutes	Hours	Minutes
0	12	0	12	0
5	12	17	11	43
10	12	35	11	25
15	12	53	11	7
20	13	13	10	47
25	13	34	10	26
30	13	56	10	4
35	14	22	9	38
40	14	51	9	9
45	15	26	8	34
50	16	9	7	51
55	17	7	6	53
60	18	30	5	30
65	21	9	2	51
66½	24	0	0	0

The preceding table shows the lengths of the longest and shortest days for different latitudes, from the equator to latitude 66½°:

(170.) Farther from the equator than 66½°, the length of the day varies from 0 to 24 hours, during the period in which the sun rises and sets at these latitudes. The number of days during which he never sets or never rises varies with the latitude. The following table gives these days for different latitudes, from 66½° to 90°:

North Latitude	The Sun does not set	The Sun does not rise
Degrees	Days	Days
66½	1	1
70	65	60
75	103	97
80	134	127
85	161	153
90	186	179

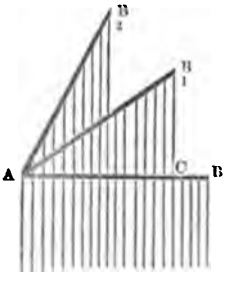
It may be observed from this Table, that the periods of light and darkness in the northern hemisphere are not equal. The reason is, that the sun appears longer in his path north of the equator than south of it. (146.)

(171.) The obliquity of the axis, at the same time that it causes a variety in the amount of light at different parts of the earth's surface, causes also a difference in the quantity of heat developed at different parts. Hence the phenomena of the seasons. Any part of the earth receives more light and more heat the longer the sun is above its horizon. During the half of the year beginning with the vernal and ending with the autumnal equinox, that is from I to III, fig. 37, the sun is longer above than below the horizon of all places in the Northern Hemisphere. The days begin to lengthen at I, reach their longest at II, and diminish again to an equality with the nights at III. At I, we in the north have the mildness of spring, at **4. The seasons.**

II, the heat of summer, at III, again the mildness of autumn. From III to I the sun is longer below the horizon than above it; at IV the night is at its longest. During this period, beginning with the autumn at III, we have the severity of winter at IV, which, at I, again softens into the mildness of spring, to renew the former course. In the southern hemisphere, from the same cause, the same variations occur at opposite times of the year.

(172.) It has recently been demonstrated* that one half of the entire sunshine which falls upon the earth does so between the parallels of 23° 44′ 40″, on each side of the equator. These parallels are only about 16′ outside of the tropics, so that the amount of heat and light in the tropical regions is about equal to that of all the other regions of the earth together. This is because the different parts of the earth are differently acted upon by the sun's rays, according as their surface is perpendicularly or obliquely exposed to these rays. Let AB, fig. 43, represent a surface perpendicular to the rays, which, owing to the distance of the sun, may be considered as coming parallel to each other. All the rays from A to B will take effect upon that surface. Let the same surface AB be now turned so as to lie obliquely to the rays like AB_1. The rays coming still in the same direction, from the same source, it is manifest that those lying between B and C are now lost to that surface and pass beyond it. In this position, therefore, it receives less light and less heat than

Fig. 43.



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* By Professor Hennessey, at the meeting of the British Association, 1862.

before; and it is evident that the more obliquely it receives the rays, as in AB_2, the less light and heat, from the same source, reach it. The sun's rays fall always perpendicularly upon some part or other between the tropics, never upon any part between them and the poles. Owing to this, and to the great absorption of heat by the air, in high latitudes, even more heat reaches the earth between the tropics, than in all other regions. And because the surfaces between the tropical circles and the latitudes of 66½°, in each hemisphere, are less oblique to the sun's rays than the surfaces between these latitudes and the poles, the former receive much more heat than the latter.

Difference of the seasons at different situations.

(173.) Places of the same latitude have thus all the same season at the same time; those of equal latitude, but in different hemispheres, have opposite seasons. Places 180° from each other in longitude have day and night opposed to each other. Our *antipodes*, therefore, that is, those dwelling on a part of the earth's surface diametrically opposite to us, having the same latitude in different hemispheres, and longitude differing from ours by 180°, have opposite seasons and opposite hours of the day from us. The dwellers on opposite points of the same parallel circle, called *perioeci*, have the same seasons but opposite times of the day; and those who inhabit places on the same meridian, but in opposite latitudes, called *antoeci*, have the same times of the day, but opposite seasons. The dwellers at the poles are both antipodes and antoeci; those on the equator but 180° from each other, are both antipodes and perioeci.

The zones.

(174.) The five divisions of the earth's surface, or zones, are named from the different effects of the sun in distributing light and heat. These are: the torrid zone; the two temperate, one north and the other south; and the two frigid. The torrid or tropical zone is contained between the tropical circles; the north temperate is contained by the north tropical and polar circles; the south temperate by the south tropical and polar circles. The north frigid zone is the part of the earth's surface enclosed by the north polar circle; the south frigid by the south polar circle. The frigid zones may be said only to have two seasons — a long sunless winter, and a continuously light, but not hot, summer. Only in the temperate zones can it be said with propriety that there are four seasons.

Double seasons at the tropics.

(175.) At the tropics, it is generally said that there is one long summer, the only form of winter being a short season of rain. It would be more strictly true to say that, between the tropics, there are two hot summers and two mild winters, during each year. It is evident that the sun, in changing his declination from *c* to *b* (see fig. 42), must, during one of his daily circles, be in the zenith of places in every latitude between the tropics. As he returns again from *b* to *c*, he will again be in the zenith at mid-day once to every place between these circles. The sun, therefore, brings what may be called mid-summer, or his time of greatest altitude, twice a year, to all these localities. At each place he is sometimes lower than the zenith, to the north, sometimes, to the south; the periods when he has sunk farthest below the zenith, on either side, may be called mid-winter. It is evident, therefore, that, to every place between the tropics, there will be two summers and two winters, the intervals between each of these varying as the place is nearer the tropical circles, or the equator. The changes of vegetation are found to correspond with these changes of the sun's declination. Some plants bloom only when he is high, others only when he is low.

It is incorrect to speak of the rainy seasons as the winter of the tropics, since these seasons generally coincide with the passage of the sun through the zenith of a place, that is, with its mid-summer.

Shadow names.

(176.) The inhabitants of the different zones are sometimes named from their mid-day shadow. Those of the torrid zone are called, on two days of the year, *ascii* (i.e. shadowless), since on the two days when the sun is in their zenith they are shadowless. The rest of the year they are named *amphiscii* (i.e. with shadows both ways), because at one time the sun throws their shadows northward, at another southward. Those of the temperate zones are *heteroscii* (i. e. differently shadowed), because at the north temperate zone, the sun is always in the south and throws the shadow northward; but at the south temperate one, the sun is always in the north, and throws the shadow southwards. Lastly, those in the frigid zones are named *periscii* (i.e. having shadows all round), because the sun, while above their horizon, throws their shadows in all directions, as he moves in a circle round them every 24 hours.

Half-hour and monthly climates.

(177.) Another division of the earth, derived from the sun's apparent arc in the heavens, is into half-hour and monthly climates. The surface of each hemisphere is divided into thirty zones. From the equator to the polar circle, where the longest day increases from 12 to 24 hours, the earth's surface is divided into 24 climates. In each of these the longest day increases by half an hour. From the polar circles to the poles, where the day increases in length from 24 hours to

half a year, the earth is divided into six climates, in each of which the longest day increases a month. The half-hour zones diminish in breadth as they recede from the equator; the monthly zones increase as they approach the poles.

TABLE OF SUCCESSIVE CLIMATES DIFFERING BY HALF-HOURS FROM THE EQUATOR TO THE POLAR CIRCLES.

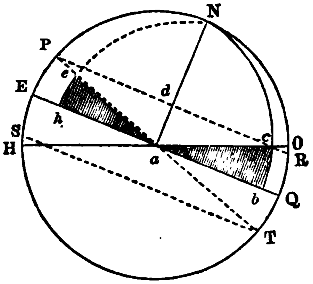
No. of climates	Latitudinal limits		Length of longest day		Breadth of the climates		No. of climates	Latitudinal limits		Length of longest day		Breadth of the climates	
	Deg.	Min.	Hrs.	Min.	Deg.	Min.		Deg.	Min.	Hrs.	Min.	Deg.	Min.
0	0	0	12	0	0	0	13	59	59	18	30	1	32
1	8	34	12	30	8	34	14	61	18	19	0	1	19
2	16	44	13	0	8	10	15	62	26	19	30	1	8
3	24	12	13	30	7	28	16	63	22	20	0	0	56
4	30	48	14	0	6	36	17	64	10	20	30	0	48
5	36	31	14	30	5	43	18	64	50	21	0	0	40
6	41	24	15	0	4	53	19	65	22	21	30	0	32
7	45	32	15	30	4	8	20	65	48	22	0	0	26
8	49	2	16	0	3	30	21	66	5	22	30	0	17
9	51	59	16	30	2	57	22	66	21	23	0	0	16
10	54	30	17	0	2	31	23	66	29	23	30	0	8
11	56	38	17	30	2	8	24	66	32	24	0	0	3
12	58	27	18	0	1	49							

TABLE OF SUCCESSIVE CLIMATES DIFFERING BY MONTHS FROM THE POLAR CIRCLES TO THE POLES.

No. of climates	Latitudinal limits		Length of longest day	Breadth of the climates		No. of climates	Latitudinal limits		Length of longest day	Breadth of the climates	
	Deg.	Min	Month	Deg.	Min.		Deg.	Min.	Months	Deg.	Min.
1	67	18	1	0	46	4	77	40	4	4	35
2	69	33	2	2	15	5	82	59	5	5	19
3	73	5	3	3	32	6	90	0	6	7	1

(178.) The boundaries of these climates may be found by the rules of Spherical Trigonometry. In fig. 44, let the circle represent the celestial sphere, HO the horizon, EQ the

Fig. 44.



Equator, PR and ST the Tropics of Cancer and Capricorn. The arc *dc* equal to *ab*, measured at N, will be the increase of the sun's half-daily arc *c* P, at the solstice, above that at the equinox, *a* E.

The increased daily arc, for each successive half-hour climate, is got by adding proportionate arcs to the semicircle described by the sun, at the equinox. Beginning from the equator, the first half-hourly climate will end where the longest day is 12½ hours. The daily arc of the sun must therefore be $180° + \frac{180°}{24}$; the increase of the whole arc is therefore $\frac{180°}{24} = 7° \, 30'$, and that of the half arc *dc*, or *ab*, 3° 45′. By this amount, therefore, will *dc*, or *ab*, for each successive half-hour climate be increased. In the triangle *acb*, *ab* therefore is known; and the angle at *a*, measured by the arc QO, is the complement of the latitude, while the arc $bc = 23\frac{1}{2}°$, the sun's greatest declination. By Napier's rules of the circular parts, $\sin ab = \cot a \times \tan bc$ $\therefore \cot a$ or $\cot \text{OQ} = \frac{\sin ab}{\tan bc}$ $\therefore$ tan NO, or the tangent of the limiting latitude of each climate$= \frac{\sin ab}{\tan bc}$;

$\therefore$ tan latitude of limit, in first climate $= \frac{\sin 3° \, 45'}{\tan 23\frac{1}{2}°}$; in second, $\frac{\sin 7° \, 30'}{\tan 23\frac{1}{2}°}$; in third, $\frac{\sin 101° \, 15'}{\tan 23\frac{1}{2}°}$, &c. &c.

The calculations of the boundaries of the other six climates depends upon the following considerations:—While the sun moves from the vernal equinox to the northern solstice, from *a* to P, fig. 44, and thus passes over 90° of the ecliptic, the boundary of the light passes from the north pole to the polar circle beyond it: it comes back again to the pole when the sun has traversed the next 90° of the ecliptic. The distance of the boundary of light from the equator depends upon the sun's declination, he (e being a point where the sun is situated in his progress round the ecliptic PT). The declination depends upon the longitude of the sun ae (52), and upon the obliquity of the ecliptic, namely eah (23½°). Now, by spherical trigonometry, $\sin he = \sin ae \times \sin eah$. If, then, we divide aP into six equal parts, each 15°, this formula will give the declinations he, at these divisions, or the boundaries of the first, second, third, &c., climates beyond the poles. By making $ae = 15°$, 30°, 45°, &c., and counting eah 23½°, these boundaries are easily found.

CHAPTER X.

THE MOON.

(179.) THE moon moves in an elliptical orbit round the earth and accompanies it in its journey round the sun. Like the earth it receives light and heat from the sun. Its greatest distance from the earth is named the apogee — its least the perigee. The mean distance of the moon from the earth is about 240,000 miles, or 60 radii of the earth. The principal phenomena of the moon, connected with its revolution round the earth, and the inclination of its orbit to the ecliptic, are as follow :—

Motions of the moon.

(*a.*) Its motions. The moon moves backward among the eastern stars, like the sun, but with greater velocity. It consequently rises, culminates, and sets later every day, for it moves in the order of the signs about the earth. On any one evening it is seen among the stars about 13° to the east of those among which it was seen the evening before.

Its changes of apparent diameter.

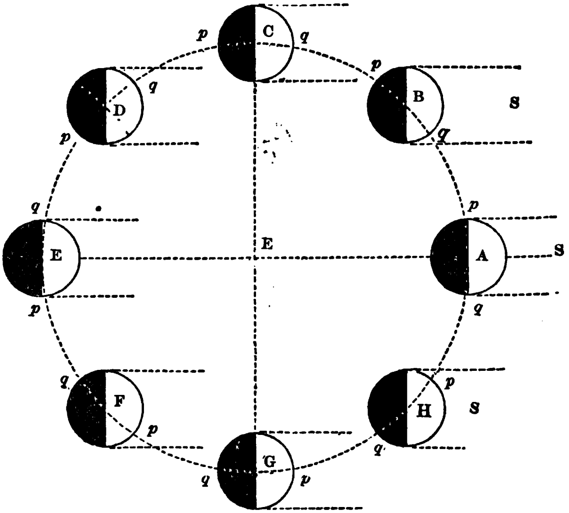
(*b.*) Its apparent magnitude. This, as well as its velocity of motion among the stars, is not always the same. It moves in an ellipse about the earth. In perigee its rate of motion is greatest, about $36' 48''$ in an hour, and its diameter is $33' 31''$. In apogee its rate of motion is least, about $29' 34''$ per hour, and its apparent diameter is only $29' 22''$. The mean of the two measurements of the apparent diameter is $31' 27''$.

Phases explained.

(*c.*) Its phases. The moon takes different shapes in different parts of its orbit; these shapes are called its phases. They repeat themselves at certain intervals. At first it is seen as the new moon in the west, shortly after sunset, in the form of a thin crescent, with its horns turned towards the left or east. In this form it sets very shortly after the sun. Next night it is a thicker crescent, and more towards the east when the sun is setting; it therefore sets later. The crescent gradually increases, as well as the eastern distance from the sun, and the moon becomes of a bright semicircular shape. This is called the first quarter. It gradually waxes larger, and at last, in about a fortnight from its first appearance, forms a full bright circle. It is now full moon. It reaches the third quarter, always receding farther eastward from the sun, in fact, now approaching it from the west, and again has the form of a semicircle, with its convex side towards the right or east. Gradually it changes into a thin crescent again, with the horns towards the right or west, then disappears for a while to reappear as new moon. All these phases are easily explained by the fact that the moon, a spherical body, revolves about the earth, and reflects the light it receives from the sun.

In fig. 45, let S be in the sun's direction from the earth E, and the moon seen in the different parts of its orbit A, B, C, D, E, F, G, H. On account of the sun's distance, its rays may be considered as falling parallel upon the moon. The explanations formerly given respecting the planet Venus will enable us to understand the moon's changes of appearance. The half of the moon upon which the sun shines is either wholly turned towards the earth, as in E, or wholly turned away, as in A,—positions which are called the moon's syzigies: or the bright side is half turned away, and half turned towards the earth, as in C and G,—positions which are

Fig. 45.



called the moon's quadratures. Or, again, the bright side is more turned towards, than from us, as in D and F; or, lastly, less towards than from us, as in B and H.

The moon, as new moon, at A, is in the same part of the heavens with the sun. She is then said to be in conjunction. She stands before him in reference to the earth, sets with him, and can therefore not be seen. At the first quadrature C, she has gradually departed 90° from the sun: at sunset, therefore, she is in the south, and remains above the horizon six hours after sunset, when she too sets. At full moon E, she has departed 180° to the east of the sun, rises therefore when he sets, and shines the whole night. In the last quarter G, when on the wane, she is 270° from him, and in fact has approached him on the other side within a distance of 90°. At this period, therefore, when the sun sets, she is still 90° below the eastern horizon. She rises six hours later, and shines on till six hours after sunrise next day.

A day or two before she disappears altogether, hidden in the sun's rays, she rises shortly before him, having approached him from the west within a very short distance; and then she may be seen in the heavens the greater part of the day with her horns turned away from the sun.

The moon's convex edge is always turned towards the sun; her concave edge, or horns, always from him. While she is waxing, she is leaving the sun behind her in the west: her horns therefore are then to the east. While she is waning, she is approaching him in the east: her horns, therefore, are then to the west.

Sidereal and synodical months.

(*d.*) Its periods. The time of the revolution of the moon from a fixed star to the same again, must be distinguished from the time of its successively reaching a certain position relative to the earth and sun — such as the time between successive new, or successive full moons. The first interval is called a sidereal, the other a synodical, month. The interval between one full moon and the next is also called a lunation.

Reason of their difference.

The sidereal month is 27 days, 7 hours, 43 minutes, 11·51 seconds: the synodical, 29 days, 12 hours, 44 minutes, 2·82 seconds. The reason why the synodical is longer than the sidereal month is, because the earth moves onward in its orbit round the sun in the same direction as the moon revolves. The moon must therefore follow the earth in its course in order to attain the same relative position between the earth and the sun. Let, in fig. 35, A, B, be two successive positions of the earth at the beginning and end of a sidereal month; let I be a fixed star always seen in the same direction, S the sun, and M, $M_{,,}$, N, the moon in different positions of its orbit. Let at first A, M, and S be all in one direction from the earth. While the moon in 27 days, 7 hours, &c., moves 360° in the heavens from M to $M_{,,}$

so as to bring the same fixed star I successively into a line with it, the earth has moved about 27° of its orbit from A to B. The sun is not now in this line, and in order that the moon and sun may again be in one line, the moon must move from M, to N. This space will require above two days to pass over it; the moon will therefore require to pass over both that, and an additional space, on account of that which the earth traverses in its orbit during these two days. This requires about two hours more, so that altogether a synodical month is about 29 days, 12 hours, 44 minutes, 2·82 seconds.

(180.) We may explain this in reference to the apparent motion of the sun. If the sun and moon be together in any point of the starry heavens, e. g. Aries, then in 27 days, &c., the moon will again be in the same point, but not in conjunction with the sun. For the sun has now advanced about 27° among the eastern stars, and about two days more must elapse before the moon comes into conjunction with the sun in a different part of the heavens. The case resembles that of the meeting of the hour and minute hands of a watch. They are together at twelve o'clock. The minute hand after an hour has revolved again to twelve, but the hour hand has advanced in its slower course, and the minute hand must revolve yet $\frac{1}{11}$th part of its circle in order to meet it.

The metonic or lunar cycle.

(181.) Twelve synodical months, or 354 days, 8 hours, 48 minutes, 38·84 seconds, make a lunar year, which is therefore about 11 days shorter than a solar year. Were the solar and lunar years equal in length, then the new moons, and the same phases, would recur on the same days of each year. But as they are always unequal, these fall upon different days each year, in such a manner that after a certain number of years the same dates recur at the times of the same phases. Suppose any year to begin with a new moon. At the end of the lunar year about 11 days of the solar year are yet to run. The first full moon of the second year must therefore fall about January 18, since 29 = 11 + 18. At the end of the second lunar year 22 days remain of the second solar year. The first full moon of the third year therefore falls on January 7, and so on; thus always on different days in successive years. But, as in 19 tropical years about 235 synodical months have passed, then the 20th year begins again with a new moon, and all the phases repeat themselves as on the same days 19 years before. This cycle of 19 years is called the Metonic, after Meton its discoverer (B.C. 433).

Calculation of Easter.

(182.) Easter Sunday, by a decree of the Council of Nice already mentioned, falls upon the Sunday following the first full moon, which occurs after the equinox (March 21). If this full moon occur upon Sunday, then the next Sunday is Easter. According to this rule, the period within which Easter may occur is from March 22 to April 25, not sooner or later. If March 21 be a full moon and a Saturday, then March 22 will be Easter Sunday, the earliest possible. But if March 20 be a full moon occurring on a Saturday, then the next full moon falls upon April 18, which in such a case must be a Sunday. Easter Sunday then is on April 25, the latest possible. The movable feasts of the Church all depend upon the time of Easter. Thus Ascension Day is always 40 days after Easter, and Whitsunday 50 days thereafter.

Age of the moon.

(183.) In order to determine the time when Easter will occur in any year, it is of importance to know the moon's age at the beginning of that year. At the end of the first year of the metonic, or lunar cycle, the moon, as may be inferred from a previous statement

(181), is 11 days old; at the end of the second, 22 days; and at the end of the third, 33 days, or one whole past lunation and about 3 days more. In the existing lunation, therefore, the moon is three days old. At the end of the fourth year it will be 14 days old, and so on. Therefore to know the age of the moon at the end of any year we require the number of that year in the lunar cycle. This is called the Golden Number. The year of Christ's birth was the first of these cycles. Hence to find the golden number for any year we must add 1 to the year of our Lord and divide by 19. The quotient will be the number of cycles which have elapsed since our era began, and the remainder will be the golden number; e. g. for 1863, we have

$$\frac{1864}{19}=98+2:$$

1863, therefore, is the second year of a cycle.

The Golden Number.

The Epact.

(184.) The age of the moon on the last day of any year usually told in even days, omitting fractions, is called the *epact*. The following table shows the golden number for each year in the cycle, and the corresponding epact till the year 1900.

Golden No.	Epacts	Golden No.	Epacts	Golden No.	Epacts
I.	0	VIII.	17	XIV.	23
II.	11	IX.	28	XV.	4
III.	22	X.	9	XVI.	15
IV.	3	XI.	20	XVII.	26
V.	14	XII.	1	XVIII.	7
VI.	25	XIII.	12	XIX.	18
VII.	6			I.	0

(185.) Had it happened that 235 lunations made exactly 19 years, then the solar and lunar relations would have exactly repeated themselves after each cycle. But 19 years contain an hour and a half more than 235 lunations, so that at the end of a cycle the moon has gone one hour and a half into the next lunation. The error amounts to a day in 16 cycles, or about 300 years. The only way to correct this would be to advance the epact one day every 300 years.

(186.) Knowing the epact we may find the moon's age for any day in any year, and thus ascertain the amount of moonlight on a given night. We add to the epact for the year the number of months that have passed since January 1; or March 1, if the date be later in the year than that. To this result we add the number of the day of the month, and if the sun be above 30, we take away the 30 and have the age of the moon. Thus for its age on February 7, 1863, we have the golden number 2, and the epact 11. To this last add one for the past month of January, and seven for the day of the month, and we have $11+1+7=19$. The moon is then 19 days old. Again, for its age on July 25, 1863, we add four for the months past since March 1, and 25 for the date, $11+4+25=40$ days, which is one whole lunation and 10 days more. We have therefore the moon 10 days old in the lunation existing at that date. For its age on any day in January we only require to add the date to the epact and diminish by 30 if necessary. It is evident therefore that the moon's age on January 31 is the epact$+1$. In 1863 the moon on that date is 12 days old. On February 28, 59 days have elapsed since the beginning of the year, that is, the time of two exact lunations. Hence, on that day, the moon's age is the same as the epact for the year. We, therefore, after March 1, count the epact from that date. Thus a sufficiently near approximation to the moon's age at all periods of the year is obtained. It cannot be quite exact, seeing that all fractions are rejected in the calculation. These rules must be somewhat modified in the case of a leap year. On February 28 the moon's age is the same as the epact; on the 29th, therefore, it equals the epact + 1. In finding the moon's age, therefore, for any period of a leap year after March 1, the epact + 1 must be added to the number of months elapsed and the number of the date.

The moon's possible height above the horizon.

(187.) The moon's nodes. The earth's path and that of the moon intersect at an angle of about 5° 8′ 47″·9. By so many degrees, therefore, is the greatest height which the moon may attain, above the greatest height which the sun may reach at any place; and by so many degrees it is occasionally, when on the meridian, lower than the sun's noon-day height. The two points in which the moon's orbit and the ecliptic intersect are called the moon's nodes. That point of the moon's orbit in which it crosses the ecliptic towards the north pole, is called the ascending node, the opposite the descending. The line joining these is called the line of the nodes. These nodes are not points fixed in the heavens, but change slowly from east to west; causing the line of the nodes to revolve about their central point nearly 3′ 10″·64 daily. A complete revolution of the nodes round the ecliptic thus occurs every 18·6 years. The interval from one ascending node to the next is shorter than a sidereal month, namely, about 27 d., 5 h., 5′ 36″, for the nodes, moving from east to west, meet, as it were, the moon in its progress from west to east.

Revolution of the nodes.

Nutation of the earth's axis.

(188.) The variations of the moon's nodes, and its consequent monthly change of place in reference to the equator, as well as to the ecliptic, recurring thus every period of 18·6 years, produce also a recurrence of the phenomena caused by the moon's attraction upon the equatorial parts of the earth. Owing to the greater amount of matter about the equator, these effects, like those of the sun's attraction, are greater there than on the other parts. The sun's greater attraction there, as we have already seen, causes the extended axis of the earth to describe a circle P P_1 P_2 . . . . (fig. 10) around the pole of the ecliptic K. The greater influence of the moon on the equatorial matter causes a similar movement of the extended axis, by which, if the movement occurred alone, the axis would describe a small ellipse round the pole of the moon's orbit in 18·6 years. But the two movements occur together: the axis of the earth goes forward round the pole of the ecliptic, and at the same time describes the ellipse round the pole of the lunar orbit, as a carriage wheel revolves on itself while it advances on the road. The consequence is that P P_1 P_2 . . . . (fig. 10) is not exactly a circle, at all times 23½° from the pole of the ecliptic, but an undulating ring. This movement round the pole of the moon's orbit is called the nutation of the axis.

The harvest moon explained.

(189.) The ecliptic is differently inclined to the horizon of different places, consequently (187) the moon's orbit will be occasionally 5° 8′·47 more, occasionally 5° 8′·47 less inclined, than the ecliptic, to any horizon. There is one consequence of this inclination to the horizon, important to the farmer in our latitudes, namely, that the moon, when near the full in the autumnal months, rises nearly at the same hour for some successive evenings. In order to explain this, let us suppose that the orbit of the sun and that of the moon are equally inclined to the horizon of any place, e. g. London— that, in fact, the moon's path is on the ecliptic.

Let the circles H H (figs. 46 and 47), represent the horizon, C Z the vertical, or, so to speak, the axis of the horizon, PP, the axis of the sphere, B D in both figures the ecliptic, and CK its axis. It is evident that while the celestial sphere turns round on PP_1 in the direction of the arrow, the equator EE turns round without changing its altitude HE. At London it forms always an angle of $90° - 51\frac{1}{2}° = 38\frac{1}{2}°$ with the horizon. Not so with the ecliptic. The axis of that circle CK turns round PP_1 and describes a cone of which C is the vertex, and the circle KK_1 the base. Hence the angle ZCK is continually varying as the sphere revolves, ranging from ZCK (fig. 46) to ZCK_1 (fig. 47). And the angle of inclination of the ecliptic to the horizon must vary exactly as their axes, namely, from DCH (fig. 46), to BCH (fig. 47). At London, the greatest inclination is $38\frac{1}{2}° + 23\frac{1}{2}° = 62°$; and the least $38\frac{1}{2}° - 23\frac{1}{2}° = 15°$.

Fig. 46.

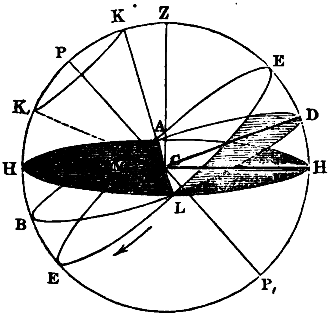
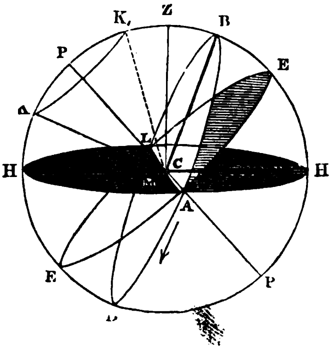


Fig. 47.



This least angle is formed when the intersection of the ecliptic and the equator, namely, the line of the equinoxes, is brought into the horizon by the daily revolution of the sphere, that is, when the line AL is in the horizon; A, the vernal equinoctial point (fig. 47), being in the east, and L in the west. Of course the vernal equinoctial point is then rising. The greatest angle is formed twelve hours after, when the line of the equinoxes is again in the horizon, but A setting in the west and L rising in the east. It is plain that the time in which any given arc of the ecliptic—say one of 13°, AM (fig. 46), or LM_1 (fig. 47)—will rise above the horizon by the revolution of the sphere, must depend upon the inclination. In fig. 46, M is less below the horizon than M_1 in fig. 47, although the arcs AM_1 LM_1 are equal. If, now, we suppose the moon to be in the part of her orbit the ecliptic, at Aries, A (fig. 46), and that she rises to-day with a fixed star at A, to-morrow she will have gone about 13° from A forward among the eastern stars and be at M; she will not, therefore, rise when A rises. But as the angle of inclination of her orbit with the horizon is then smallest, the extremity of the arc of 13°, M, is not much below the horizon when A rises, and the moon will again become visible very little more than 24 hours after her last rising. On the contrary, when the moon is rising in that part of its orbit opposite to Aries—namely, Libra, L (fig. 47)—the inclination of the ecliptic to the horizon is great. It rises at a given hour to-night, with L; next night it is 13° eastward; and when L, exactly 24 hours after, rises again, the moon is still far below the horizon, though, as before, only about 13° along the ecliptic from L. In fact, when the moon is in Aries, her arc of 13° would only delay her rising for 17 minutes each day; but when she is in Libra, the same arc causes a delay in rising, from day to day, of 1 hour and 17 minutes. Every month this occurs, because every month she passes through these two points of her orbit. But the differences are not noticeable every month; they strike most when occurring at the period of full moon. If the moon be in the part of her orbit where her times of rising differ least, she must be in Aries; and if she be full at the time, the sun must be setting as she rises, and directly opposite to her, namely, in Libra. But the sun is in Libra only in autumn: hence it is that we see the autumn full moon rising evening after evening about the same hour.

The explanation applied to the actual orbit of the moon:

(190.) All the above is true of the moon as supposed to move in the ecliptic. But she moves in an orbit inclined to the ecliptic more than 5°. She is therefore sometimes more oblique to the horizon than the ecliptic is, by above 5°, sometimes less so by the same. The consequence of this is, that the differences of the times of her rising are sometimes less than 17 minutes, and sometimes more than 1 hour and 17 minutes on successive evenings.

and to an orbit on the equinoctial.

(191.) As the equinoctial never changes its angle in reference to the horizon, it is evident that, if the moon's orbit were on it, her 13° eastward would be described uniformly, and she would rise about 50 minutes later each day, being always about the same distance below the horizon at every interval of 24 hours from her last rising. She would also culminate 50 minutes later. A lunar, like a solar day, is thus longer than a sidereal, and for the same reason (145).

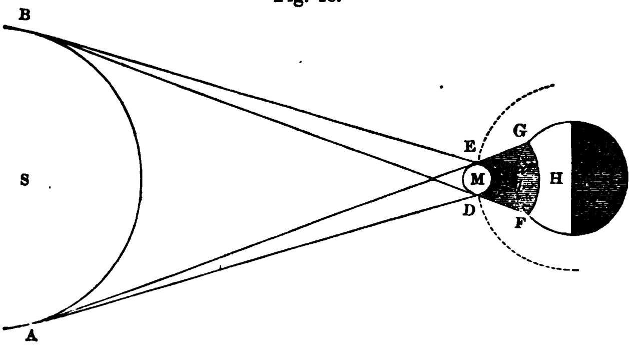
Eclipses not always when the moon and sun are in opposition.

(192.) Were the moon's orbit coincident with the ecliptic, then at every period of new moon there should be an eclipse of the sun, and at every period of full moon an eclipse of the moon. For at both periods the centres of the earth, sun, and moon, would be in one straight line. At new moon (see fig. 45) the moon would be directly between the earth and the sun, and would therefore screen or cover the sun from the earth. At full moon, the earth, being directly between the sun and the moon, would throw its shadow over the moon. But since the orbits intersect, although it is still true that eclipses of the sun can only happen at the time of new moon, and eclipses of the moon only at the time of full moon, still they do not happen every time of new and full moon. They occur only when new or full moon takes place, with the moon in one of its nodes, or very near it. When it is not in one of these, then the new moon goes wholly above or wholly below the ecliptic, at such a distance as prevents it from covering the face of the sun; and the full moon goes wholly above or below the ecliptic, in such a manner that it completely avoids the shadow of the earth.

An eclipse of the sun explained.

(193.) Eclipses of the sun are total, partial, or annular; of the moon, total or partial. Total and annular eclipses of either are also said to be central, when the observer's place on the earth is in the same straight line with the centres of the sun and moon. Let the circle round S (fig. 48) represent the sun, M the moon, and the circle round H the earth; then D*ba*E is the incomplete conical shadow of the moon, or that space within which no light from the sun can enter, owing to the interposition of the moon. *a*EG and *b*DF represent the penumbra or space surrounding the central shadow, where only a part of the sun's rays can enter, and so much the fewer of them the nearer the part of space is

Fig. 48.



to the edges EG and DE. From EG and DF, therefore, all round the central shadow, a want of light prevails, changing from full sunlight to the deep shadow of the conical space. If any parts of the earth come within the deep shadow as the surface from *a* to *b*, there will be at these parts a total eclipse of the sun, because the moon covers the whole disc of the sun to the inhabitants of these parts, and suffers none of its light to reach them. To the dwellers midway between *a* and *b* under the axis of the conical shadow, the eclipse will be central as well as total. The parts of the earth between G and F will have a partial eclipse of the sun, and so much the less of it will be eclipsed the nearer the place is to EG or DF, because so much less of the moon will be directly between that locality and the sun. The moon moves in her orbit from west to east. An observer at F will see the eastern edge of the moon D touch the western edge B of the sun, and the eclipse will begin for him. But at the same instant the spectator at G sees the western edge E of the moon quit the eastern edge A of the sun, and the eclipse just ending; so that, in the circumstances here supposed, all the parts between G and F have a partial eclipse, except *ab*, which has a total eclipse.

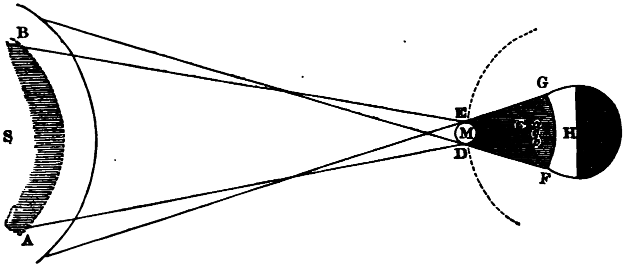
A total eclipse.

(194.) But it sometimes happens that the earth and the moon are farther apart than the length of the axis of the cone which the moon's shadow, if uninterrupted, would form. In this case some of the parts of the earth turned towards the sun, although not touched by the shadow, may be directly in the line of the axis of the cone, prolonged as the point *c* (fig. 49). If we prolong the sides of the cone from its vertex C, the base of the new formed cone will meet the earth and intercept a certain portion of its surface *ab*. From all the earth within this portion the moon will be seen as a dark

An annular eclipse.

circle, covering the middle of the face of the sun, and leaving the rest as a bright ring — namely, the parts nearer the circumference of the sun than the points B and A. This is called an annular eclipse; and thus eclipses are either total or annular, according as the distance of the earth from the moon is less or greater.

Fig. 49.

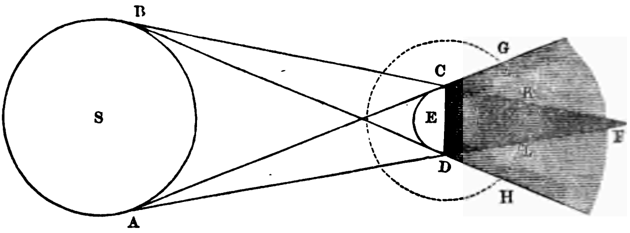


Both the total and the annular begin and end as partial eclipses.

An eclipse of the moon explained.

(195.) To explain an eclipse of the moon, let the circle about S (fig. 50) represent the sun, E the earth, and the dotted circle about E the moon's orbit; then DCF is the conical shadow of the earth through which the moon must be passing when it is eclipsed. Such an

Fig. 50.



eclipse is total or partial, according as the moon passes wholly or only in part through the shadow of the earth; for in the one case the shadow covers the whole, in the other only part, of the moon's disc. At the instant when the central point of the moon is in the axis of the cone of the earth's shadow, a total eclipse is also central.

Until the eastern edge of the moon is at the point G of its orbit round the earth, she will be fully illuminated, receiving rays from the whole of the sun's surface; but whenever she enters the penumbra by crossing the line AG, the earth will cut off a part of the sunlight, and she will receive less and less as she passes on toward K. After crossing CF in K she is totally eclipsed, and remains so till she reaches L. On crossing DF in L she emerges again into the penumbra, and gradually as she moves on receives more light from the sun, till she finally passes out of the penumbra on crossing the line BH, and again receives the full measure of his rays.

Eclipses of the moon have the same appearance at the same time, wherever visible.

(196.) When an eclipse of the moon occurs, the moon is actually deprived by the earth of the light it would otherwise have received from the sun. An eclipse of the moon must then be the same for all places which at the time have the moon above their horizon. It will, therefore, be visible at all these places at the same time, and be of the same kind, total or partial. In an eclipse of the sun, that orb is not really deprived of its light, since it is itself the source of light; but the parts of the earth which have the moon directly between them and the sun are deprived of the sun's rays, by the moon arresting them in their progress to the earth. This eclipse, as already explained, is seen by observers at one part of the earth leaving the eastern edge of the sun, at the instant when to others it is beginning on the western edge; at the same instant, to those between them the eclipse is either total or annular, and to others at the side of the deep shadow, but not out of the penumbra, more or less of the sun is hid; others, again, outside of the penumbra can, beyond the edge of the new moon, see the whole disc of the sun. Thus an eclipse of the sun is at the same time total, partial, or altogether invisible to different places. In the calculation of longitudes from eclipses, the indistinctness of the edge of the earth's shadow on the moon renders the precise moment of the beginning and ending of a lunar eclipse not easily ascertainable: hence for this purpose they are less useful than eclipses of the sun. In these the circle of the moon is always distinct; the calculations which they require are however much more complicated.

An eclipse of the sun differs to different observers at the same time.

The Chaldaic cycle.

(197.) We have already (181) mentioned the metonic cycle in which the dates of the moon's phases recur. A different cycle (though some have confounded them) is that called the Chaldaic. This is a period of about 18 years 11 days, in which the moon returns to the same position with regard to its nodes, therefore to the same latitude, and eclipses must, therefore, recur in the same order, and of the same magnitude, as in the preceding interval of the same length. For 223 of the moon's mean synodical revolutions occur in 6585·32 days, while 19 complete synodical revolutions of the nodes occur in 6585·78 days. It is evident that the moon can have changed but little from this point in regard to its nodes at the end of the 19th year; so that the period of nineteen years, or the cycle of phases, is also, very nearly, the cycle of eclipses.

The moon turns on its axis in the same time as it com-

(198.) Everyone is familiar with the indistinct likeness to a human face, visible on the moon's surface. As viewed from the earth the appearance is always the same;—the same marks are always seen, because the moon has always the same side turned towards the earth. It is a necessary consequence of this that the moon must revolve on its own axis, and that so as to fulfil one of these revolutions in the same time as it completes one revolution round the earth. Were it not so, we should see different sides of the moon at different times, and it should, therefore, present a different appearance at different times. In fig. 35, let the circles round A and B represent the moon in two different parts of its orbit, and S the earth. If the moon were to proceed from A to B without revolving on its axis, then the radius AM would remain parallel to itself, and at B would have the direction BM_1, and the lunar spot M, which, from S, was seen in the centre of the moon at A, would at B be seen near the edge, on the point M_1. But the same mark is always seen in the centre. The moon therefore, in moving from A to B, must revolve on an axis perpendicular to its orbit in such a manner as to give the radius AM the new direction BN—namely, through the angle M_1BN. But $M_1BN = ASB$, the angle described by the moon, at the earth, during its revolution from A to B. When, therefore, the moon has gone round the whole of its orbit, or 360°, it must also have turned 360° on its axis, or once completely round.

pletes its orbit.

(199.) The important phenomena of the tides is a result of the united attraction of the sun and moon on the waters of the sea. The influence of the moon in the formation of the tides is greater than that of the sun. The essential facts respecting them are as follow:—

The tides.

1st. Twice daily, a rise and fall of the sea occur on every shore, named flood and ebb. These succeed each other at intervals of about six hours.

Five facts respecting them.

2nd. The farther east a place lies, the earlier on a given day does this phenomenon occur.

3rd. At each locality the floods and ebbs occur about fifty minutes later every 24 hours.

4th. At the periods of the moon's syzigies the floods are higher, and the ebbs lower, than at her quadratures. So, too, the rush of water is greater when the moon is in her perigee, and the earth in perihelion, than when these bodies are severally in apogee and aphelion.

5th. Local hindrances to the free motion of the waters, such as islands, capes, straits, narrow estuaries, cause delay in the times of flood and ebb.

We give the explanation of these facts in the order of their statement.

Explanation of the first fact;

(200.) 1st. Since the sun is about thirty millions of times the size of the moon, and about 400 times farther from the earth than the moon is, we may, by the aid of the law of gravitation, compare their relative attraction on the centre of the earth. Attraction is directly as the mass, and inversely as the square of the distance. The ratio, therefore, of the sun's attraction to that of the moon is as $\frac{30,000,000}{400^2} : 1$; that is, as $\frac{30,000,000}{160,000} : 1$; or as $195 : 1$ nearly. The earth's *centre* is thus attracted about 195 times more by the sun than by the moon. But it does not follow from this that the sun should be most effective in causing tides: for these depend not upon the *amount* of attraction exercised by the sun or moon. It is upon the *difference* between the attraction which each of these bodies exerts upon the centre, and that which it exerts upon the surface of the earth, that the tides depend. This difference is greatest in the case of the moon, and therefore its effect on the tides is greatest.

The water on the surface of the earth next to the sun or moon is drawn with greater force by either, than the centre of the earth is drawn. That on the surface turned away from them is drawn with less force than the centre. On the first side there is a rising up of the waters towards these bodies, on the other a holding back compared with the central point of the earth. Thus on both sides of the earth there is a rise of the waters from the centre, which causes two high tides diametrically opposite to each other. This, of course, draws the waters from the other parts of the earth's surface, so that, at the same time, there are two low or ebb tides opposed to each other, and at right angles to the existing high tides. Were the attraction of the sun and moon equally exerted all over the earth, then these ebb and flood tides could not exist. Deductions from the law of gravitation show that the moon's attraction at the earth's *surface* above which she is moving is $\frac{1}{30}$th greater than her attraction at the *centre* of the earth. That of the sun is only $\frac{1}{12000}$th greater at the surface than at the centre. The whole attraction of the moon is $\frac{1}{198}$th that of the sun at the centre. But $\frac{1}{198} \times \frac{1}{30}$ is more than twice greater than $\frac{1}{12000}$, and therefore the influence of the moon is more than twice that of the sun in raising the tides.

of the second;

(201.) 2nd. By the earth's rotation from west to east, any point upon the earth's surface comes earlier under the moon the more it lies to the east.

of the third;

(202.) 3rd. The earth, as before shown, must turn somewhat further than a sidereal revolution in order to bring the moon again on to the same meridian, because she moves in her orbit while the earth revolves (191). For this further turning about 50 minutes is required.

of the fourth;

(203.) 4th. At the syzigies or periods of conjunction and opposition, the lunar flood tides correspond with the solar, and the lunar ebbs with the solar ebbs; so that each works with the other, and the highest spring tides, as they are called, occur. At the quadratures, the lunar floods coincide with the solar ebbs, and the lunar ebbs with the solar floods; they thus counteract each other and make neap tides. So when the earth is in perihelion and the moon in perigee, as it and the sun are then nearest the earth, the floods are greater than when these are at their greatest distances in aphelion and apogee.

of the fifth. (204.) 5th. The origin of all the tides is in the Southern Pacific and Antarctic Oceans. No other parts of the sea afford scope enough for the formation of a perfect tide, since this manifestly requires an expanse of water such that, when the sun and moon are both in the The tide wav ; its origin meridian above its centre, the fluid should be free to flow towards that centre from 90° on each side of it. The Atlantic does not average above 50° in breadth. The South Pacific, stretching from Cape Horn to Australia, and communicating by the Antarctic with the Indian Ocean and the Atlantic, affords the largest continuous mass of water, and in this the tide is formed. The spring tide elevation of the water takes place there, shortly after the sun and moon have passed the meridian together,—*after*, because of the inertia of the water. This mass of water tends to follow the sun and moon westward, and to come more directly under them northward, they being on the average near the equator. It thus becomes a wave pressing northward and westward, pushing before it other masses of water, and raising them into a wave. The tides in all small seas and estuaries are due to this wave being driven into them, any direct influence which the sun and moon may have in raising tides in these smaller seas being imperceptible compared with the influence of the immense tide wave. This wave requires time for its propagation: it is delayed in its progress by islands, shoals, &c., and anged in its course by projecting headlands. It moves onward from the South Pacific, passing partly round the south side of Australia, and partly, by the north side, through the straits of the eastern archipelago. It then winds round the east coast of Africa, doubles the Cape of Good Hope, and presses northward along the Atlantic, and is lost in the Arctic Sea. Had the parts of the earth near the equator been covered with one continuous sea, it is plain that this wave would have followed the moon and sun, and set a girdle round the earth in 24 hours; that is, with a mean velocity of above 1,000 miles an hour. But it requires more than 12 hours to pass up the Atlantic; so that it moves through it at about 600 miles an hour. and course.

Establishment of a port.

(205.) The obstacles which delay the tide wave being always the same, act always with the same amount of hindrance; and as the causes which first raise the tides act uniformly, the local tides must follow each other in regular and equal periods. At different ports observation is made of the interval between the passing of the sun together with the moon over the meridian and the time of high water, because the tides follow in regular periods dependent upon that interval. This is called the 'establishment of the port.'

Cotidal lines.

(206.) The manner in which the tide wave travels is discovered by ascertaining a number of places, on the sea and along coasts, at which the hour of high water is the same on a given day in Greenwich time. By drawing lines through these points on a map, for successive hours, we are enabled to trace the movements of the crest of this mighty wave. These lines are called cotidal lines. Such a map shows that the wave raised at 12 o'clock midnight off Van Diemen's Land, travelling onward, causes high water next noon at Cape Comorin. Then dashing on the eastern coast of Africa it approaches the Cape of Good Hope, and by next midnight its crest is stretching across the Atlantic from Cape Blanco to Newfoundland. Part of the main wave before next noon has disappeared in the Arctic Ocean. Another part, turned from its course by Scandinavia, proceeds southwards through the North Sea, and meets with a branch of the Atlantic tide which has worked its way through the English Channel. Thus the tide wave is 12 hours old when off the Cape of Good Hope, 24 hours at Newfoundland, 36 hours old when it reaches Aberdeen, having passed round the north of Scotland, and 48 hours old when it carries the merchandise of the world into the port of London. The branch which enters the Thames by the English Channel, however, is only 36 hours old.

Age of the tides.

(207.) By observing the day and hour of the spring-tide next after the conjunction or opposition of the sun and moon, we may learn the age of the tide at any place.

Tidal bores.

Where two shores form a strait, the tides between them are delayed and successive tide waves approach each other. Thus the waters become heaped up in the narrow space. At the Bay of Fundy the tide rises sometimes 120 feet; whereas in the open sea, at St. Helena, its height is about four or five feet. Sometimes one tide arrives at the lower part of a river before the previous one has reached its highest point; and a third may approach the mouth before either of the former has fulfilled its course. The ebb of the first tides meeting with the flow of the later ones causes what is called a bore; that is, a sudden and violent wave rushing up the river. In the Amazon this wave is often 180 feet high. Within 200 miles of its mouth, eight tides are advancing at once up this river, and often as many as five bores are in progress at the same time.

Atmospheric tides.

(208.) The action of the sun and moon produces tides in the atmosphere as well as in the ocean. The appreciable effect of these tides, however, is extremely small.

CHAPTER XI.

REFRACTION AND TWILIGHT.

(209.) It has been seen already that the aberration of light causes the apparent place of a star to differ from its real place in the heavens, and, unless allowed for, becomes a source of error in observations. The amount of deviation, however, from this source is small, and its exact amount easily ascertainable. But there is another source of error, in the change which rays of light undergo in passing through the atmosphere, in which the deviation of the apparent from the true place is greater, and its exact amount less easily appreciable.

Rays of light bent by the atmosphere.

A ray of light passing from any medium of transmission, such as air, into one denser, such as water, is bent *towards* the perpendicular on the surface of the denser medium. Or, if it pass from a dense into a rare medium, e. g. from water into air, it is bent *from* the same perpendicular. The same bending of the ray occurs, if it pass from a rare stratum of air into a denser, or the reverse. Now, as air becomes gradually denser near the surface of the earth than it is higher up, light in passing through it is continually bent as it progresses; so that, instead of reaching the earth in straight lines, the latter part of its course from any heavenly body is curvilinear. The ray thus bent is said to be refracted. An object is seen in the direction which the light has on entering the eye, that is, along a tangent to the curve formed by the ray at the eye. Hence the effect of refraction is evidently to make the apparent height of the heavenly bodies greater than the actual height. In figure 51, if S be the actual position of the sun's centre below the horizon S_1AC^1, it will be seen at S_1 on the horizon by the observer at A. This is owing to the bending of the ray in passing through the 45 miles stratum of atmosphere, bounded by A′B′C′B″, which floats above and round the earth ABC. The law of refraction is, that the more obliquely a ray falls upon any medium the more it is bent; that is, the angle of refraction increases as the angle of incidence. Rays of light fall more obliquely on the atmosphere from a heavenly body on or near the horizon than from one higher up; from a body in the zenith they fall perpendicularly, and are not refracted at all. Hence atmospherical refraction is greatest when the object is in the horizon: its amount then is about 33′ or 34′. Consequently when the sun is seen on the horizon it is 33′ higher than its true place in the heavens. Its own apparent diameter is about 32′. In the morning, therefore, when its upper edge is first coming above the plane of the horizon, we see the whole of it up, and in the evening, after its whole orb is actually below that plane, we see it just beginning to dip. The length of the day, as given in the table (169), should be increased by twice the time which the sun requires to move through 33′. At the equinoxes daylight is thus prolonged about 7 minutes, at the solstices about 9.

Consequently, the length of the day is increased.

(210.) The sun rises to the observer at the north pole, not when he is on the equator, but when he has yet 33′ of south declination.

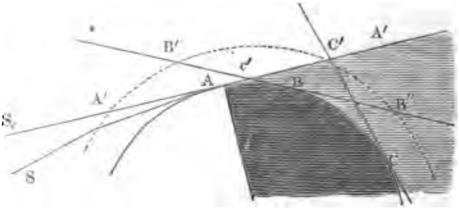
(211.) At an altitude of 45° the angular amount of refraction is 57½″ and at the zenith 0. Tables of the refraction for all altitudes have been drawn up, which afford means of correction for every observation.

(212.) When the sun or moon are just above the horizon, their diameter perpendicular to the horizon is affected by the refraction differently at its two extremities. The upper extremity is less refracted than the lower: hence the lower part of the orb is more lifted up than the upper, and the consequence is that the sun and moon in that position assume a somewhat oval shape.

Twilight explained.

(213.) Refraction, we have seen, lengthens the period of light by actually, as it were, detaining the sun above the horizon; but even after it has finally disappeared, the atmosphere still sends us light received from it: so that, even after sunset, we have that pale and lessening light known as twilight. This is owing, not to refraction, but to the reflection and scattering of the sun's light by the atmospheric vapours, and perhaps by the particles of the atmosphere itself. In figure 51, it is evident that when the sun is 33′ below the horizon S_1AC^1 its direct rays can no longer reach the earth at A. But if the circle A′B′C′B″ represent the outer edge of the atmosphere surrounding the earth ABC, its rays, just after it has disappeared, will still fall on the whole of the atmospheric segment A′B′C′, and that will reflect the light to an observer at A. At the same time an observer

Fig. 51.



at B, a place further east, has also a little light reflected from the particles of the atmosphere in the segment B′C′B″, although the sun has set to him before it set at A; and to an observer at C, still further east, the last glimmer of twilight has just departed, for the departing ray SC′ of the sun is just on the point where his western horizon cuts the upper edge of the atmosphere. Twilight ceases at any place when the sun is 18° below the western horizon, and in the same way dawn begins when he is yet 18° below the eastern horizon—a larger and larger segment of the atmosphere receiving his light, till at last he appears himself.

Duration of twilight at different latitudes.

(214.) It is easy to estimate the duration of twilight. At the equator it is shorter than in north or south latitudes, because the sphere there being right (see fig. 42) the sun descends perpendicularly upon the horizon, and describes an arc of 18° in 1 hour 12 minutes, namely: $\frac{18^\circ \times 24}{360^\circ}$.

At any middle latitude, the time which the sun requires to sink 18° varies with the latitude and the declination (see figures 38, 39, and 40).

In all latitudes above $48\frac{1}{2}^\circ$, the sun's position at midnight on a midsummer's eve is less than 18° below the horizon, and twilight lasts all night. For example: at the summer solstice the sun is distant from the pole $66\frac{1}{2}^\circ$; but at London the pole is $51\frac{1}{2}^\circ$ above the horizon. Therefore the sun can only be $66\frac{1}{2}^\circ - 51\frac{1}{2}^\circ = 15^\circ$ below the horizon of London at midnight.

CHAPTER XII.

THE TERRESTRIAL GLOBE.

(215.) On the ordinary terrestrial globes, a number of the more important points and circles are marked, such as the equator, the parallels for every 10° or 15° of latitude, both tropical circles, both poles, and meridians for every 10° or 15°, and also the ecliptic. Surrounding the globe is a brass circle graduated, which serves as a fixed meridian, and within which the globe revolves. Close to the globe under the north polar point is fixed a small brass circle, which revolves with the globe, and is divided into 24 equal parts corresponding to the hours of the day. This circle may be turned round so as to bring any hour under the brass meridian, and the edge of the brass meridian serves as an index to show whatever part of the hour-circle is brought under it.

Description of the artificial globe.

Surrounding the globe, in a plane perpendicular to that of the brass meridian, is a wooden circle, representing the rational horizon. This is divided into several concentric circles, one marking the degrees from 0 to 90°, between the points east, west, north, and south; another showing the 32 points of the compass; another the signs of the zodiac in order; another the degrees in these signs; another the months corresponding to these signs and degrees; and another the days.

Beneath the wooden horizon a mariner's compass is often placed; and accompanying such a globe should be a detached thin brass quadrant, which can be fixed by means of a screw at any part of the brass meridian. This last is called the quadrant of altitude. It is graduated upwards from 0 to 90°, and downwards from 0 to 18°, for use in problems connected with twilight.

As every place marked can be brought under the brass circle by merely turning the globe round, this circle may be made to represent the meridian of any place. And when the place is brought under it, the latitude of that place is found marked on the meridian, which, beginning with 0 at the equator, is graduated to 90° at the poles. So, too, in the degree marked on the equator where the brass meridian intersects it, we find the longitude of the place.

When, by the turning of the globe, the hour circle moves, so that, instead of the division at first under the meridian, the next division is brought there, the artificial globe has passed through as many degrees of its revolution as the earth does in one hour. The divisions on the wooden horizon form a perpetual calendar, by which we may find the celestial sign, and degree of it, through which the sun on any given day is passing, and by means of this his declination for that day.

Problems with the equator for horizon.

(216.) The solutions of problems in mathematical geography by means of an artificial globe are, at best, but approximations to correctness, and can never supersede the necessity of mathematical calculations. Still the globe aids one in realising the idea of the thing sought in the problem, and, when great correctness is not required, may be used with advantage. Many problems require only a knowledge of how to set the globe for their solution. Some of these are so simple, that we shall set them down in the form of questions without any answer. The following are worked by setting the globe so that the equator coincides with the wooden horizon;—

1. How is the latitude and longitude of a place ascertained?

2. How is a place found whose latitude and longitude are given?

3. How is the difference of latitude and longitude of two places found?

4. How is the difference of local time at two places found from their longitudes?

5. What places have mid-day at the same instant as any given place?

6. What o'clock is it at A when it is mid-day at B?

Solve this by the hour-circle, and without it by the longitudes of the two places, as marked on the equator.

7. What o'clock is it at A when it is any other hour than 12 at B — e.g. 2 o'clock?

8. What places of the earth have mid-day when the place A has any given hour?

Bring A under the brass meridian, set the given hour on the hour-circle also under the meridian, and then turn the globe till the hour of 12 come under it. The places now under the brass meridian are those required. But it should be observed that if the given hour be before noon, only places to the east of A can have mid-day; the globe, therefore, must be turned from right to left. If the given hour be after noon, only places to the west of A can have mid-day of that day, and the globe should be turned from left to right.

9. How do we find the periœci, antipodes, and antœci of any place?

Bring the given place under the meridian. The point of the brass meridian on the other side of the pole, and equally distant with the given place from the wooden horizon, on the same side of that horizon as the given place, gives the locality of the periœci.

The place diametrically opposite to the given place is that of the antipodes; and that which is equally distant from the wooden horizon, on the same side of it with the antipodes, but on the other half of the meridian, gives the place of the antœci.

10. How do we determine the distance of two places from each other?

Bring one of the places under the brass meridian. Screw the quadrant of altitude to the point of that meridian immediately above the place. Stretch the quadrant over the globe to the other place, and read the distance in degrees as marked on the quadrant. Should the quadrant be too short, then find the antipodes of the first place, and the distance of that as before from the second place. What that distance lacks of 180°, is the distance of the two given places in degrees.

Multiply the distance thus found by 60 for geographical, or 69½ for English, miles.

Problems with globe set differently.

(217.) The following problems require an appropriate setting of the globe for their solution.

1. How do we set the globe according to the polar height of any place, and so that the directions of places, as marked on the artificial globe, shall correspond to the directions of the actual places on the earth?

The first is done by means of the divisions of the brass meridian. We incline the axis of the globe till that degree of the meridian, as reckoned from the pole, which expresses the latitude of the place, touches the wooden horizon. Thus, for London 51½° should touch it.

The second is done by means of the compass, when the frame of the globe is so turned that the needle stands above the line which expresses its deviation from the north line at our locality. This deviation is called the variation of the compass.

2. In what direction does the place A lie from the place B?

Set the globe according to the polar height of the place B. Bring B under the brass meridian. Screw the quadrant of altitude over B and stretch it over A to the wooden horizon: there the direction is given.

3. How do we find the time of the rising and setting of the sun, and hence the length of the day at a given place, on a given day?

Set the globe according to the polar height of the place. Find in the calendar on the wooden horizon the position of the sun on the given day. Determine that position on the ecliptic; bring the point thus found under the meridian, and bring XII. of the hour-circle also under the meridian. Turn the globe first eastwards till the point of the ecliptic reaches the wooden horizon, then similarly westwards. In the first case the hour on the hour-circle under the meridian tells the time of rising; in the second, the hour of setting: whence the length of the day is determined.

4. How do we find the length of the day in the last exercise, as increased by morning and evening twilight?

Screw the quadrant of altitude on the brass meridian over the latitude of the given place, and lay it eastwards over the globe. Then proceed as in the last exercise, but continue the motion of the globe eastward till the sun's place coincides with 18° (213) on the quadrant of altitude below the horizon; the hour under the brass circle shows the first moment of dawn. Screw the quadrant of altitude again over the same latitude, and lay it westward. Then turn the globe westward till the sun's place on the ecliptic be 18° under the western horizon: the hour under the brass meridian now shows the last moment of twilight.

5. How do we find the altitude of the sun when place, day, and hour are given?

(*a.*) For mid-day altitude, set the globe to the polar height of the place. Bring the point of the ecliptic in which the sun is on that day under the meridian, then read on the meridian the number of degrees from the wooden horizon to that point.

(*b.*) For altitude at any other hour. The globe being set as before, bring the place of the sun again under the meridian. Bring the hour XII. also under it. Turn the globe to the right or left according as the hour is A.M. or P.M. until the given hour comes under the meridian. Then read on the quadrant of altitude, screwed at the zenith and stretched to the horizon, the altitude of the sun's position on the ecliptic.

How is the problem worked when the conditions are changed: namely, to determine the hour when place, day, and altitude are given?

6. In what manner is the earth enlightened by the sun on a given day and hour?

Raise the pole above the horizon as far as the declination of the sun on the given day if it be northern, and sink it as many degrees below if the declination be southern. Turn the pole exactly to the horizon if the declination be nil. In other words, set the globe so that the place of the sun, found by means of the calendar, may be in the zenith. The hemisphere above the horizon gives the day side of the earth, that below, the night side. Therefore, the places under the meridian have mid-day, those east of it afternoon, those west forenoon, at the same absolute time; and each place has so much later or earlier hours the further they lie on one side or other of the meridian. If we wish to know the parts of the earth enlightened at a certain hour of a given place, then set the globe so that the sun's place may be in the zenith, and bring the place under the brass meridian. The upper hemisphere shows the day side of the earth at mid-day of the given place. Bring the hour XII. under the brass meridian, and turn the globe till the given hour comes under the meridian; then the problem is worked.

The highest place under the meridian is that which has the sun in the zenith at the given hour.

7. How do we find the parts of the earth in which an eclipse of the moon is visible, when the time of the central eclipse at a given place is known?

Find the place where the sun is in the zenith at the time of the central eclipse. The antipodes of this has the moon in its zenith, since the moon and sun are in opposition at the time of a lunar eclipse. Turn the globe so that this antipodes be in the zenith; then at all places above the horizon the eclipse is visible. The places under the brass meridian have the moon eclipsed at the given hour in their meridian; those on the western horizon see it rising; those on the eastern see it setting.

CHAPTER XIII.

MAPS AND PROJECTIONS.

(218.) ALTHOUGH the solutions of problems in mathematical geography by means of a globe give only an approximation towards correctness, still an artificial globe is useful, not only for the greater ease with which, by means of it, many problems are solved, but also because it affords the only means we have of faithfully representing to the eye the relative positions of the different places on the earth's surface.

Maps represent the earth incorrectly.

(219.) But artificial globes are inconvenient as well as expensive. Hence we have recourse to maps in the study of geography. It is evident that the plane surface of the paper on which a map is drawn can never be made truly to represent the curved surface of the earth, and distortion must occur in some form or other. To reduce this distortion as much as possible, or to cause it to take place in some way unimportant for the special purposes of the student, is the object of the different methods which have been devised of constructing maps.

Five different kinds of projections.

(220.) Of these methods we shall describe five, commonly called projections. They are the orthographic, the stereographic, the globular or equidistant, the conical, and Mercator's projections.

The orthographic described.

(221.) In the orthographic, the eye of a spectator is supposed to be at an infinite distance from the earth, viewing the surface of a hemisphere as if it were all on a flat surface, passing through the centre of the earth. This surface is the plane of projection. We may form an idea of this projection by supposing the earth to be a hollow sphere and the crust transparent, so that rays of light falling parallel on it would pass through it. Were these to fall upon a sheet of paper equal in area to a large circle of the globe, passing through the earth's centre and lying so that the rays might fall upon it perpendicularly, then the photographic picture of the earth's surface which they might be imagined to form is exactly the representation of the earth which the orthographic projection gives.

The stereographic described.

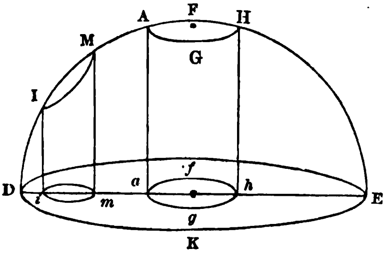
(222.) In the stereographic projection, the eye is supposed to be on the surface of one hemisphere, and looking inwards through the transparent globe to see the opposite hemisphere drawn, as it were, on an immense sheet of glass passing through the centre of the earth, just as we see the objects outside of a window apparently drawn upon the panes.

The globular described.

(223.) In the globular, the nature of the imaginary drawing is the same as in the stereographic, but the eye is supposed to be at a distance beyond the surface of the opposite hemisphere, equal to the length of the sine of an angle of 45° on the earth's sphere.

(224.) In fig. 52, let the hemispherical surface represented by DFE be projected, orthographically, on the plane DEK. It is evident that a circle on the sphere parallel to the plane of projection will be represented by a circle in that plane.

Fig. 52.



Thus GAFH becomes *gafh*. But a circle or curve whose plane is perpendicular to the plane of projection, becomes a straight line in the representation. Thus the curve DFE becomes the straight line DE. A circle on the sphere oblique to the plane of projection becomes an ellipse. Thus IM becomes *im*.

Defects of the orthographic.

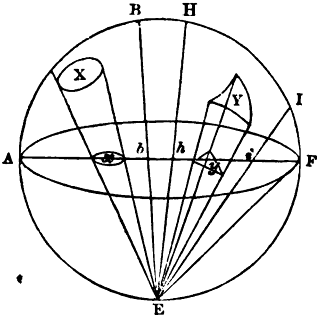
(225.) It is also evident that the lengths in the projection are less in proportion as they proceed from the centre to the circumference; thus the length DI is projected into D*i*, while the equal length AH is projected into *ah*.

The parts near the centre, therefore, of a hemisphere projected orthographically are represented nearly in their true proportions; those near the circumference are distorted and diminished. For this reason the orthographic projection is not fitted to represent large portions of the globe.

Advantages and defects of the stereographic.

(226.) Let the hemispherical surface ABF (fig. 53) be projected stereographically on the plane ADF. The eye in that case is supposed, from its position at E, to see the hemisphere ABF from within, as if it were through and therefore upon the transparent plane ADF. In this projection, as in the last, every circle or curve whose plane is perpendicular to the plane of projection becomes a straight line; thus ABHF becomes A*bh*F. But the stereographic possesses an advantage over the orthographic, in that every other circle than one in the plane of the eye becomes a circle in the projection.

Fig. 53.



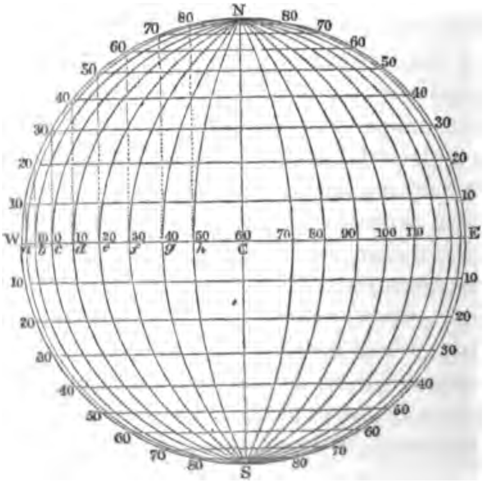
Thus X becomes *x*. So, too, every spherical triangle on the surface becomes a similar plane triangle in the projection. Thus Y becomes *y*. The spaces in this projection are enlarged beyond their due proportion in proceeding from the centre to the circumference. Thus BH is represented by *bh*, while the equal arc IF is represented by *i*F greater than *bh*. At the circumference, therefore, of a hemisphere thus projected, the parts drawn are magnified and somewhat distorted; though the distortion is less than in the orthographic.

(227.) The manner in which disproportion or distortion occurs in the other projections will be sufficiently evident from their method of construction. We now proceed to describe this for all the projections.

Construction of the orthographic on the plane of a meridian.

(228.) Orthographic projection of the sphere on the plane of a meridian.

Fig. 54.



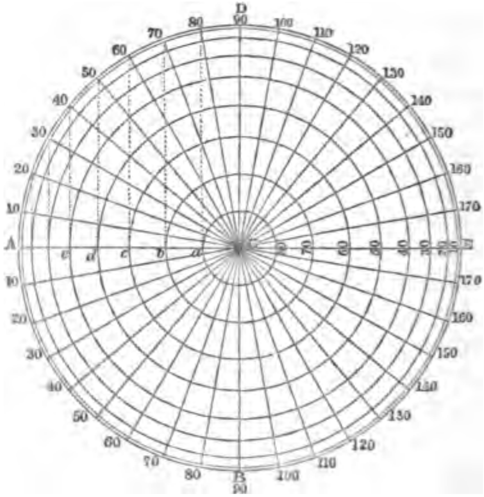
Draw the circle NESW (fig. 54) for the meridian bounding the plane of the projection. Draw the diameters NCS, WCE, perpendicular to each other. WCE is the equator projected, NCS the earth's axis. The quadrants NE, ES, SQ, QN, being each marked off into arcs of 10°, join the divisions by the parallels 10-10, 20-20, &c. These are projections of the parallels of latitude for every 10 degrees; and the parallel circles being perpendicular to the plane of projection, the parallels are projected into straight lines. At $23\frac{1}{2}°$ on each side of WE the tropics may now be similarly drawn, and at $66\frac{1}{2}°$ the polar circles. Let fall perpendiculars upon the equator from the points 80, 70, 60, &c., of the quadrant NW; it is manifest that these perpendiculars will cut the radius WC in the same points as perpendicular rays of light from the bulging quadrant of which WC is the projection falling from points on that quadrant at the same distances from W as 80, 70, 60, &c. are. Let these perpendiculars cut the equator in *a*, *b*, *c*, *d*, &c. The arcs of circles which shall pass through the two poles, and through these points, will give the projection of the meridians for every degree of longitude. In the same way the points, *a*, *b*, *c*, &c., may be found for the half of the equator CE, and the projections of the meridians drawn. We may now assume any of these meridians to be the first, and number each successive line from it, 10, 20, 30, &c.

Construction of the orthographic on the plane of the equator.

(229.) Orthographical projection of the sphere on the plane of the equator—that is, on the horizon of the pole. A circle DABE (fig. 55) is first described for the equator, and two diameters drawn, DB, AE. The quadrants are divided as before. Then draw diameters from the points of division in the one quadrant to the corresponding points of the quadrant opposite; and these diameters will be the projections of the meridians. Assuming any one of them AC as the first, number them from 0 to 180 on each side of AC.

Let fall perpendiculars from 10, 20, 30, &c., on AC, meeting it at the points *a*, *b*, *c*, &c., and from the centre C through these points describe circles. These circles will be the projections of the parallels, and may be numbered 10, 20, 30, &c., beginning from the circumference or equator.

Fig. 55.

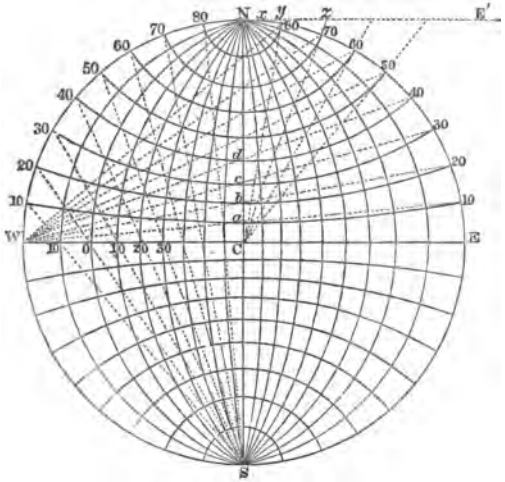


The distortion and diminution consequent upon drawing the outline of each locality within its proper lines of longitude and latitude, is evident from the diminution of the squares towards the equator in this figure, and towards the boundary meridian in figure 54.

Construction of the stereographic on the plane of a meridian. (230.) Stereographic projection of the sphere on the plane of a meridian.

Describe the circle NESW (fig. 56) for the boundary meridian. A diameter NCS will represent the central meridian projected. Another, perpendicular to it, WCE, will be the equator. Divide and number the quadrants as in fig. 54. It is evident that rays from the projected surface NESW to the eye opposite to

Fig. 56.



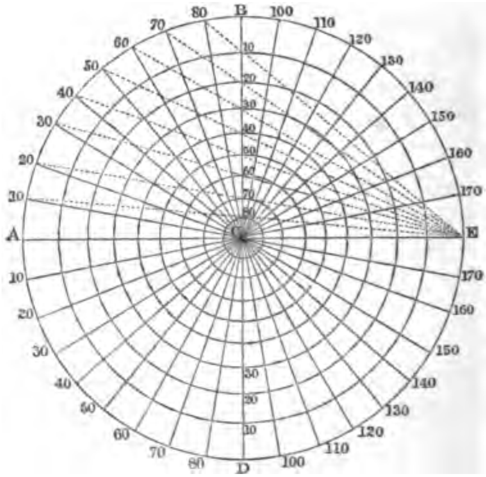
C and distant from it by the length of the radius, in a direction perpendicular to the page at the point C, will cut the radius WC in exactly the same points as those in which, if we suppose the eye at S, rays from points of the circumference ENW would cut it. Draw, therefore, from S the lines S10, S20, S30, &c. The intersections of these, with WC, give the points through which the projected meridians must pass. The arcs of circles passing through these points and the two poles will be the projected meridians. For a similar reason, draw from W the lines W80, W70, &c., then intersections with NC give the points through which the curves of the parallels must pass. The points 10, *a*, 10; 20, *b*, 20; 30, *c*, 30, &c., are thus three points in each parallel circle; the curves may therefore be drawn (Euc. iv. 5.) Assuming the meridian of 20° W. for the plane of projection, and finishing the figure, the framework of the usual map of the eastern hemisphere is constructed.

Construction of the stereographic on the plane of the equator.

(231.) Stereographic projection of the sphere on the plane of the equator, or horizon of the pole.

Describe the circle BADE (fig. 57) for the equator.

Fig. 57.

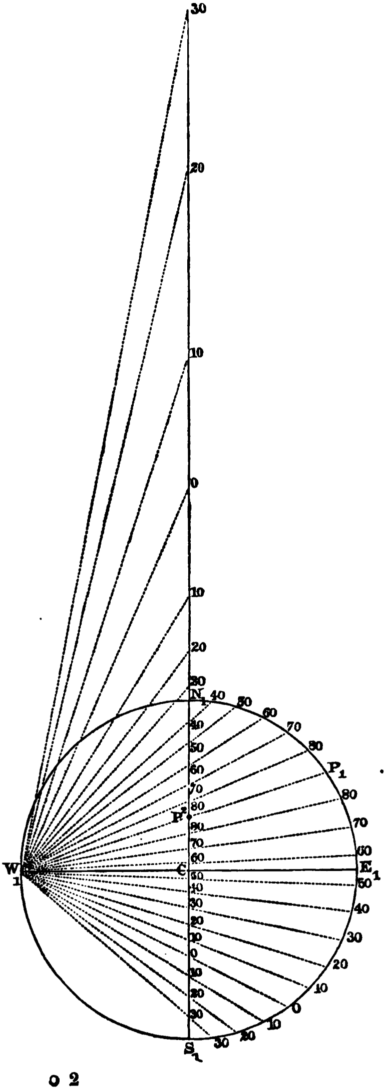


Draw, as before, two diameters, AE, BD, at right angles to each other, and divide the quadrants. From the points 10, 20, 30, &c., in the quadrant AB, draw lines to E, which point is similarly situated with respect to the radius BC, as the eye is, in its supposed position at the end of the earth's radius opposite to C (i. e. towards the reader). The intersections, 80, 70, 60, 50, 40, &c., of these lines with BC give the points through which the parallels must be described as circles, with C for their common centre. The divisions 10, 20, 30, . . . 180, in the circumference, being joined to the centre, will be the projections of the meridians.

Construction of the stereographic on the horizon of London.

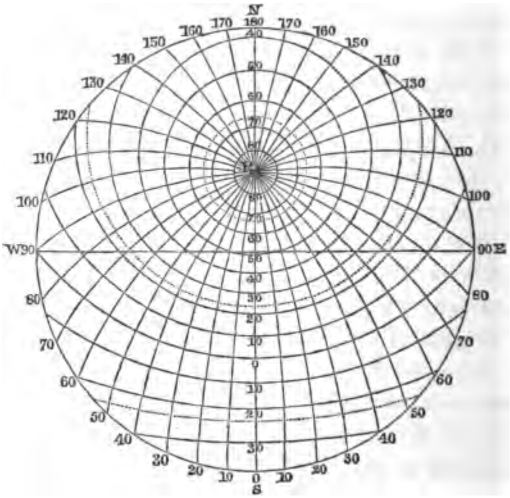
(232.) Stereographic projection of the sphere on the plane of the horizon of London.

Fig. 58.



In this projection the eye is supposed to be placed at the antipodes of London (a little south-east of New Zealand), and viewing the opposite hemisphere from within, in such a way that London occupies the centre of the field of view. As this projection is more complicated than those before mentioned, we shall, for convenience, take two circles of equal radius, in one of which $N_1W_1S_1E_1$, fig. 58, the essential points for the projection may be found; and in the other NWSE, fig. 59, the lines which make the complete figure may be inserted from measurements found by means of the first circle. Draw at right angles the diameters N_1S_1, W_1E_1, and NS, WE, in their respective circles. N_1S_1 and NS will be the projections of the meridian at right angles to the eye, that is, the meridian passing through London, and $N_1S_1W_1E_1$, NSWE, in each figure will be the north, south, west, and east points of the horizon of London. The point W_1 is in respect to the diameter N_1S_1 the same as the supposed position of the eye opposite to the point C. As the north polar point at London is 51½° above the horizon, measured on the meridian, measure (in fig. 58) from N_1 to P_1 51½°, and join W_1P_1. The projection of the pole will be the point in which W_1P_1 intersects N_1S_1, namely in P^2. Measure along the circumference of the horizon in both ways from P, arcs of 10°, and draw lines from W_1 to the points thus found. The intersections of these lines with N_1S_1 will give points

Fig. 59.*



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* To avoid confusion of lines, the circle in fig. 59 has been drawn of larger radius than that in fig. 58, although they are described in the text as of equal radius, and in actual construction must necessarily be so.

through which the different parallels of latitude must pass. Number these points along N_1S_1 or N_1S_1 produced, the same as their corresponding points on the circumference. The line 80, 80, extended on either side of P^2, will be the diameter of the parallel circle of 80°. The distances of these two points from N_1 may now be measured and transferred to NS in figure 59, and the parallel circle 80, 80, described by bisecting the line 80, 80 to find the centre. So 70, 70 will be the extremes of the 70th parallel, and may be marked off in fig. 59. Similarly, the 60th, 50th, 40th, &c., parallel circles may all be described. The whole of the five first circles is within the circle of the finished projection (fig. 59), since the pole is more than 50° above the horizon of London. Of the circles described with 30-30, 20-20, 10-10, 0-0, 10-10, 20-20, 30-30 for diameters, it is evident that only part will be in the finished projection. By finding points $23\frac{1}{2}°$ on either side of P_1 (fig. 58), and joining them to W; a diameter for the projection of the north polar circle may be found. Points $66\frac{1}{2}°$ on either side of P will afford means of finding a projection of the northern tropical circle which is not wholly above the horizon. It is evident that the equator must pass through the east and west points of the horizon, and through the projection of a point 90° from P_1 (namely O), that is, in fig. 58 it will pass through the points WE, and intersect N_1S_1 in O. Points in the circumference $E_1N_1S_1W_1$, $90° + 23\frac{1}{2}°$ on either side of P, will lead to the projection of the part of the southern tropic included within the horizon of London.

(233.) In regard to the drawing of the meridians, NS the meridian passing through London is already drawn. The part of that meridian in the projection, lying between the point of projection of the pole and the circumference, is counted as the first meridian and numbered O; the remaining part, extending from the pole to the opposite point of the circumference, is the 180 meridian line. Let the semicircles on each side of NS in fig. 59 be divided into arcs of 10°, and numbered each way from 0 at S to 180 at N. Now every meridian must pass through the pole, and its projection therefore through the projection of the pole. In the points 10, P, and 170 to the right of N, we have therefore three points through which the meridian next to NS on one side must pass; the circle may therefore be described. So with the points 20, P, 160 for the next again; 30, P, 150 for the one after; and so on. Thus the projections of all the meridians may be drawn.

The globular or equidistant projection.

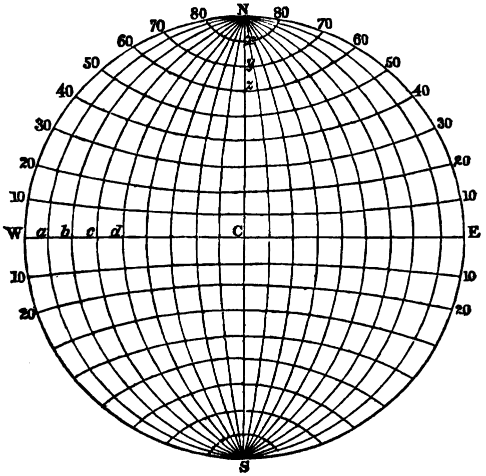
(234.) The stereographic and orthographic projections do not represent equal spaces on the globe by equal spaces in the projection. The orthographic diminishes spaces below their true proportions; as they approach the circumference of the projection; the stereographic magnifies the same above their true proportion. In the orthographic the eye is at an infinite distance from the surface projected; in the stereographic, it is supposed to be exactly on the circumference of the earth. Some station between these two positions of the eye has been sought, for a point in which the projection would represent equal spaces on the globe by equal on the map. This object is *nearly* attained in the projection which supposes the eye to be at a distance from the earth equal to the sine of 45° on the circumference of one of its great circles. Such a projection is called the globular. In constructing it, those spaces on the plane which represent equal spaces on the globe are often made *exactly* equal to each other. In this case it is named the equidistant projection.

Construction of the equidistant on the plane of the meridian

(235.) Equidistant projection of the sphere on the plane of a meridian.

Describe the circle NESW (fig. 60), for the meridian, whose plane is that of the projection. Draw two diameters at right angles to each other; WCE for the equator, NCS for a central meridian. Divide the quadrants each into nine equal parts of 10°. Divide also each of the radii NC, WC, SC, EC into nine equal parts. Number the divisions of the quadrants from the equator to each pole 10, 20, 30, &c. Name the divisions of the central meridian, *x*, *y*, *z*, &c., those on the equator *a*, *b*, *c*, *d*, &c. For the parallels, we have now only to describe circles, for each of which three points are given, namely, 80, *x*, 80—70, *y*, 70, &c.; the centres of these circles are all in the prolongation of the line NS. For the meridians, circles have to be drawn similarly through the points N*a*S, N*b*S, &c., the centres of which are all in the prolongation of the line WE.

Fig. 60.

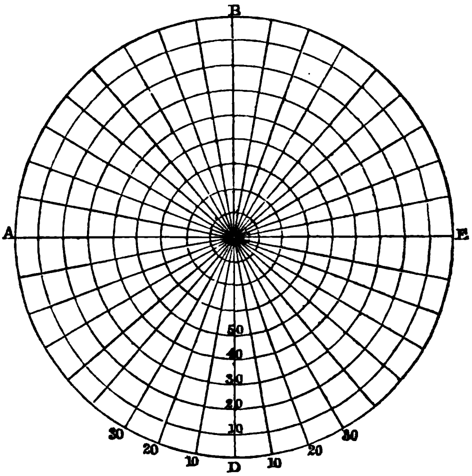


Construction of the equidistant on the plane of the equator.

(236.) Equidistant projection of the sphere on the plane of the equator.

Describe a circle BADE (fig. 61), for the equator. Draw BD, AE at right angles to each other. Divide the quadrants as before, and one radius CD in the points 10, 20, 30, 40, &c. Then from the point C, which is the projection of the pole, through 10, 20, 30, &c., describe circles concentric with the equator: these will represent the parallels of latitude. The projections of the meridians will be found by drawing straight lines from the divisions on any quadrant through the centre C, to the divisions of the opposite quadrant.

Fig. 61.



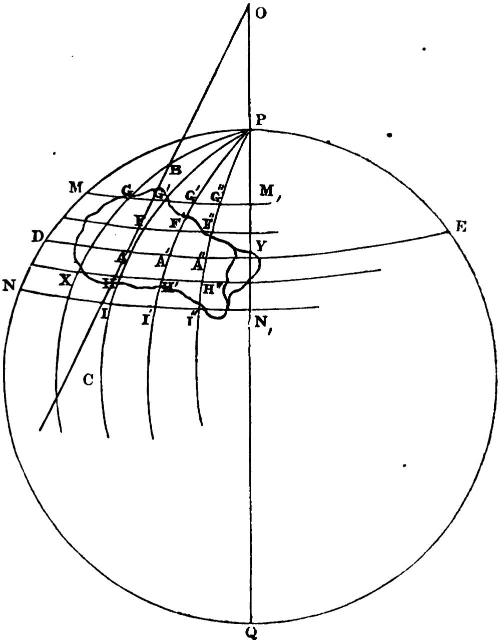
(237.) The equidistant projection is that commonly employed in the construction of maps of the world. Its defect, as may be seen from the figures, is that, although equal spaces on the globe are represented in this projection by equal spaces, and, therefore, the proportionate *size* of different countries is preserved by it;

still lines, such as the parallels and meridians, which cross each other at right angles on the sphere, cross at different angles in the projection. Therefore, the *shape* of the countries represented must be distorted. The projection on the plane of the equator exaggerates the dimensions of countries near the equator in longitude; for it is evident that the degrees on the equator and parallels near it are much larger than the degrees of the meridians.

The conical projection.

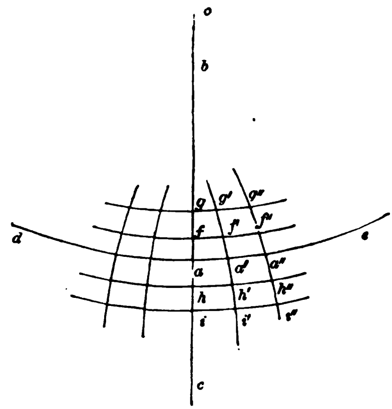
(238.) The conical projection or developement. The general principle of this method is as follows: Let PAC (fig. 62) represent a meridian, PQ the axis of the

Fig. 62.



earth, and A a point on its surface. Suppose, now, that the line AO, being a tangent to the earth at A, and forming an angle with the axis at O, revolves round that axis, it will evidently describe a cone, which coincides with the sphere along the parallel circle DAE. If, again, we suppose the hollow cone thus described to be opened out, the parallel circle on the sphere will be represented by an arc of a circle in the developed cone, described about O as a centre, with the radius OA, and the parts of the spherical surface on the circle will be identical with the parts of the conical surface along that arc. But the parallel circles MG and NI, supposed to be transferred from the spherical to the conical surface, must evidently be larger on the cone than on the sphere, and the parts about them magnified. The difference, however, will be trifling, when a narrow zone of the sphere MM$_1$N$_1$N only a few degrees on each side of DA is transferred to NMM$_1$N$_1$ on the cone.

Fig. 63.



Construction of the conical projection.

(239.) Let it be required to represent the country XY (fig. 62) on a conical developement. We choose first a meridian BAC, and a parallel DAE, both of which pass very nearly through the middle of XY. The middle meridian BAC is represented on the developement (fig. 63), by the straight line *bac*. To represent the middle parallel, which should pass through the middle *a* of the plan, we must find the length of a tangent to the sphere on the middle meridian, that is, of the line AO (fig. 62). This line is evidently the tangent of the colatitude of the central point A, or, which is the same thing, the cotangent of the latitude. We transfer (from a scale of parts) the length of AO to *ao* along the straight line *bac*, then from the point *o*, as a centre, describe the arc *dae*. This arc represents the middle parallel in the developement. In order to find the places of the other parallels, we measure (using the same scale) along *bac* the distances *af*, *ag*, *ah*, *ai*, &c., respectively proportional to the lengths AF, AG, AH, AI, &c. These distances may be either 1° or 10° of the sphere, or any other number, and will vary in length, as degrees of latitude vary at different distances from the equator (p. 67). Arcs of circles described from the point *o* as a centre through the points *f*, *g*, *h*, *i*, will give the projection of the parallels.

(240.) To represent the meridians we measure on the parallels now found arcs *aa′ aa″*; *ff′ ff″*; *gg′*, *gg″*; *hh′*, *hh″*, &c., equal (by the scale of parts) respectively to the arcs on the globe AA′, AA″; FF′, FF″; GG′, GG″; HH′, HH″, &c. The lengths of these last are known from a table of the lengths of degrees of longitude in different latitudes (p. 53). Lastly, we trace through the points thus obtained the curves *g′f′a′h′i′*, *g″f″a″h″i″*, &c., and these represent the meridians on the sphere transferred to the cone, namely, G′A′F′H′I′, G″A″F″H″I″, &c.

(241.) In this system of developement there is but little distortion of the form or exaggeration of the extent of any small part of the globe. If we compare the part of the earth's surface F′G′F″G″ (fig. 62), with the developement of that part in fig. 63, we see that on the globe the meridians and parallels all intersect at right angles; and we therefore regard $F'G'F''G''$ as a small rectangle whose base is $F'F''$ and height $F'G'$. On the developement this portion is not a rectangle, because the meridians and parallels do not intersect at right angles; but it may be regarded as a parallelogram, whose base is $f'f''=F'F''$, and height $f'g'=F'G'$. The rectangle $F'G'F''G''$, and the parallelogram $f'g'f''g''$ are, therefore, equal in extent of surface, having the same base and equal altitudes. Hence it follows that the different parts of the country XY, when laid down in their proper latitudes and longitudes on the developement, are represented as each having the same extent in proportion to the other in developement that they have on the surface of the earth. The distortion is but little, for the angle at which the meridians cut the parallels differs but little from a right angle, in the developement of a small part of the earth's surface. Such distortion will occur principally towards the edges of the map.

Projection of a map of England.

(242.) In projecting a map of England by this method, the meridian of 2° west longitude is chosen as a central meridian, and the 53rd parallel of latitude as the central parallel. On each side of the 53rd parallel, two equal spaces may then be marked off along the central meridian at such distances as will suit the scale of our intended map, for the points through which the 52nd, 51st, and 54th, 55th parallels must pass. Each of the spaces laid off will represent a distance of about 69·16 miles, being 1°. The length of the radius of the parallels which should pass through these points, *ao* (fig. 62), is now to be found in an extension of the central meridian. It is evidently the cotangent of 53°. This may be found in terms of the radius in the ordinary trigonometrical tables, but we may express it in degrees of latitude, by remembering that the radius of a circle

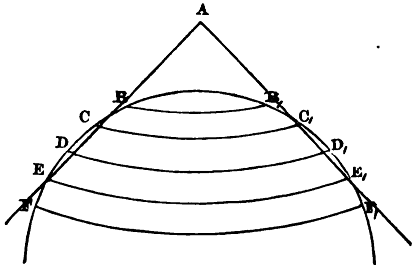
$=57^\circ{\cdot}295$ of its circumference in length. Therefore, $\cot.\ 53^\circ \times 57^\circ.295 = {\cdot}753554 \times 57^\circ.295 = 43^\circ.175$. The length allotted for 1°, in our map, must then be extended 43·175 times along the central meridian to find the centre from which to describe the parallels.

(243.) We have thus found the central meridian and the parallels of latitude; the points in which the other meridians intersect these parallels will be found by measuring, along each parallel, distances which will represent the number of miles in 1° of longitude on that parallel. These distances must of course be laid down in the same proportions as the distance of 69·16 for a degree is laid down on the central meridian. The points for four meridians on each side of the central one are all that is required to include the whole of England and Wales in one projection.

Delisle's modification of the conical.

(244.) An ingenious adaptation of the conical method was invented by Delisle for a map of Russia. The side of the cone AF (fig. 64) is not made a tangent, but intersects the surface of the sphere in two points exactly in the middle between the central and the extreme parallels. The surface of the cone and that of the sphere thus coincide along two parallels CC, and EE,, while the intermediate parallel DD,, is somewhat short, and the two extremes BB, and FF, somewhat long, in the conical developement. The errors are thus distributed over the map; but in a representation of a small part of the

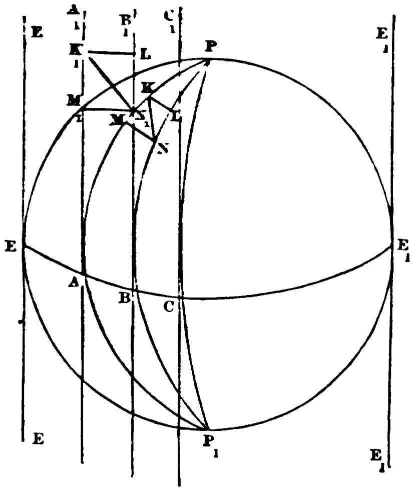
Fig. 64.



world, the outline thus obtained may be considered as almost exact.

Mercator's projection described.

Fig. 65.



(245.) Mercator's projection is a representation of the surface of the earth on that of a hollow cylinder circumscribing the sphere of the earth. In fig. 65 let the lines EEE, $E_1E_1E_1$, represent the sides of a circumscribing cylinder, and PEP_1 an artificial globe divided into compartments or *gores*, bounded by equidistant meridian lines PAP_1, PBP_1, PCP_1, &c. Let the surface of this globe be supposed perfectly elastic. Every part of the circumference of the equator EABCE will be in contact with the inner surface of the cylinder. If now we suppose that the half gore PAB is detached from the surface of the globe, and lifted up so as to be applied to the cylindrical band AB A_1B_1, in order that it should cover the whole of that band, the gore must be stretched and enlarged at every point except on the equator, and stretches the more the farther the point is from the equator. It is evident that by such treatment the form of the different parts on the surface of the sphere must be changed: for if we take any very small part of the spherical surface KLMN, and suppose it to be transferred to $K_1L_1M_1N_1$, we necessarily lengthen the parallels KL and MN. There is but one way of preventing the change of form, that is, to lengthen in the same proportion the lines KM and LN. By this means the figure $K_1L_1M_1N_1$ on the cylinder will preserve the same shape as KLMN on the sphere, since, being very small, they may both be considered as rectangles having their bases and heights proportional. Each half gore and the objects on it being similarly transferred to the surface of the cylinder, with the meridians lengthened in proportion as the parallels on the sphere contract towards the pole, the parts of the sphere near the poles when transferred to the cylinder will necessarily be at a very great distance from the equator on the cylinder, and in fact the polar points will be infinitely distant. We shall thus have obtained a representation of the whole earth on the surface of a cylinder, which will extend to an infinite distance above and below the equator. If now we suppose the cylinder to be slit open along one of the meridians and stretched out, we have a Mercator's chart.

(246.) In the chart thus opened out the equator becomes a definite straight line; the meridians infinite straight lines perpendicular to the equator, and the parallels straight lines parallel to the equator, but not equidistant from each other. The higher the latitudes the greater the distances by which they are separated, in consequence of the rapid diminution of the parallels on the sphere, and lengthening therefore of the meridians on the cylinder.

Table of meridional parts.

(247.) We may find the rate of increase in length of the meridians on the cylinder, by considering that, if we divide a meridian on the globe into minutes of a degree, one of these minutes at any latitude will be to a minute of longitude on the same parallel as the radius of the equator to the cosine of the latitude (78), that is (by plane trigonometry), as the secant of the latitude to the radius of the equator. In the chart, therefore, the length of 1′ of the meridian, at any parallel, must exceed that of 1′ of the longitude at the same, in the proportion which the secant of the latitude bears to the radius of the equator. But the minutes of longitude are all exaggerated (except on the equator), since every parallel is represented on a straight line equal to the equator; much more, then, must the meridians be exaggerated in length. If, now, we take 1′ of longitude on the equator as the unit of measurement on our chart, and assume that as radius, we have—rad. : sec. of lat. = 1′ of long. : 1′ of lat. in a given parallel, in which proportion the first and third terms are equal, and therefore the second and fourth will be so. We thus find that a table of natural secants to radius 1′ of the equator, will give us 1′ of latitude in any parallel, and, by adding together the secants of every 60′, we can have the whole length of 1° of latitude in the cylindrical projection, nearly enough for practical purposes. The numbers obtained by adding together the secants of 1′, 2′, 3′, &c., to the last minute of the meridian, counted from the equator, form what is called a table of meridional parts.

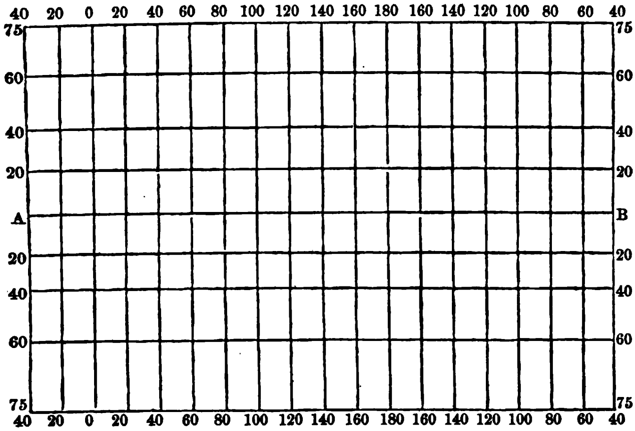
(248.) The following table shows the length of the enlarged meridian for every 5 degrees of latitude, on the supposition that the earth is a perfect sphere.

Latitude	Meridional parts	Latitude	Meridional parts
0°	0·00	50°	3474·47
5	300·38	55	3967·97
10	603·07	60	4527·37
15	910·46	65	5178·81
20	1225·14	70	5965·92
25	1549·99	75	6970·34
30	1888·38	80	8375·20
35	2244·29	85	10764·62
40	2622·69	90	Infinite
45	3029·94		

Construction of Mercator's projection.

(249.) To construct the chart, take a line AB (fig. 66), of the length required for the equator. Divide it into 36 equal parts, so that the meridians may be 10° apart. (In figure 66 they are drawn 20° apart for the sake of convenience.) Through the divisions draw the meridians perpendicular to AB. To find the place of the parallel of 10°, look opposite that number on the table of meridional parts; and there it will be found that the number of these parts between the equator and that parallel are 603. Now, as the equator has been divided into spaces of 10°, each of these spaces will contain 600 minutes of longitude or meridional parts. Therefore mark off the length of one of these divisions on the equator, and 3 parts more, on the two outer meridians, and join by a line the points thus marked off; that line will represent the parallel of 10°. So to find the 20th parallel, the table of meridional parts shows that from the equator to it is 1,225 parts, a space equal to two of the divisions along the equator and 25 parts more. These measurements will be most

Fig. 66.



easily made by having a scale of parts, in which a space equal to the divisions on the equator represents 600 parts. The other parallels on both sides of the equator may all be found in the same way. Nothing now remains but to select any meridian as the first; then number the meridians at the top and bottom of the chart east and west from the first. Number also the parallels along the meridians at the sides, north and south of the equator. The outlines of the different continents may then be inserted in their proper latitudes and longitudes.

Great circle sailing.

(250.) Evidently this chart is altogether useless for showing the relative extent of different parts of the globe, that is, as a map for the study of geography. But as a chart, that is, as a map for the use of navigators, it amply atones for its deficiencies in other respects. It has been shown (76) that the shortest distance between two points upon the earth is the arc of a great circle which joins these points. By this, then, if possible, a navigator should sail from port to port. In sailing due north or south, along a meridian, he is going on a great circle, and if the two ports lie on the same meridian, he is taking the shortest way. So, too, in sailing east or west along the equator, he is taking the shortest distance between two places on that line. But except on a meridian or on the equator, it is impossible to sail along the arc of a great circle without constantly changing the direction of the vessel. It is evident that the only great circle which makes always the same angle with the meridians is the equator; every other great circle cuts them at different angles. To keep on any other great circle a navigator would require constantly to be changing the course of his vessel, that is, setting it at different angles to the meridians (which tend always north and south) as he crosses them. This is a work too difficult and complicated for most navigators. When the place of departure and that to which he is bound lie in different latitudes and longitudes, what he requires is to know some *one direction*, by constantly sailing in which, he may reach his destination; that is, he wishes to go between the two points, and always cut the meridians at the same angle. If he knows this direction, then he has only to see by the compass that he keeps in it; and though the curved line which he thus follows is longer than the arc of a great circle between the two places, the ease in sailing counterbalances the extra length of the voyage. The line along which a vessel sails, always cutting the meridians at the same angle, is called a rhumb line, or loxodromy.

Rhumb lines.

(251.) It is easy to see what form such a line on the sphere will take in Mercator's chart. Any small portion of that line, between two meridians very near each other, may be considered as the diagonal LM (fig. 65) of a small rectangle KLMN. The part LM of the loxodromy will become L_1M_1 on the cylindrical developement; and since the rectangle $K_1L_1M_1N_1$ is similar to the rectangle KLMN, the diagonal will make with the meridian K_1M_1 on the cylinder an angle equal to that it makes with KM on the sphere. The curve, then, which meets all the meridians on the sphere at the same angle, will also meet all the meridians on the cylinder at that angle. But the meridians on the developed cylinder are all parallel lines; and as only a straight line can cut parallels with the same angle, it is evident that the loxodromy, in Mercator's projection, becomes a straight line. We can now see the use of this chart to the navigator. Having marked on the chart the present position of his vessel and the place to which he is bound, he draws a line between these points, and the angle which that line makes with the meridians on the chart is the direction in which he must steer. The facility in guiding himself thus obtained is more advantage than the saving of distance by sailing on a great circle. Not that the difference between the two routes is inconsiderable. To sail by a great circle from Land's End to Newfoundland, both in latitude 50°, and differing in longitude by 48½°, the distance is 2,141 miles. Along the parallel of 50° it is 50 miles longer. On an arc of a great circle, Lizard Point is 3,178 miles from Bermuda; by the loxodromy it is 3,243, or 65 miles more. Tables have been published to encourage the practice of great circle sailing. They show the necessary changes of direction; but the method is wanting in the simplicity of Mercator's.

Map drawing.

(252.) The projections give the framework of a map. By drawing every point of the sea-coast or other boundary of any island or country in its appropriate place of latitude and longitude on this framework, the correct outline of that country is obtained. Next, by marking the windings of rivers with lines passing through the same latitudes and longitudes on the map as the actual rivers do on the globe; and by placing marks for towns in their appropriate latitudes and longitudes, we obtain a map of a country as referred to the sea level. But something more is wanted in a good map, namely, the *relief* of the surface, that is, marks which will give some idea how the land lies above the sea level. There are various ways of representing hills, mountains, and table lands, but the general mode is merely to give a sort of picturesque effect, by deeper or lighter shades for higher or lower mountains, according to the taste or whim of the artist.

But it is evident that for the purposes of the surveyor, and railway or artillery engineer, maps must, if possible, give the slopes and levels of the surface mathematically correct. In order to effect this, mathematical science has occasionally been applied to this branch of geography, by methods of which our limits can only afford a brief account.

Contour line maps.

1. Contour line maps. These are maps upon which lines are drawn joining all parts which have the same height above the sea level. The line expressing the coast line of the country must evidently be one, and the lowest, of these lines. Lines joining all parts that are 50 feet; then 100, then 150, 200, and so on, may be inserted after the coast line is drawn; or any other scale of altitudes may be used according to convenience. Such lines are simply projections of what the coast line of a country would be should the sea by any convulsion rise to the height of 50, 100, 150, &c., feet above its present level. It is evident that, where a country slopes gradually, these lines will be widely apart; where it rises steeply, they will be close upon each other. A part dark with lines will therefore show mountains; parts very light will express plains and valleys. By introducing appropriate tinting into the belts between the curves, the relative elevations of a country may be very effectively shown.

Shaded maps.

2. Shaded maps. In these shading is introduced by lines between the contour curves, so as to combine, if possible, picturesque effect with mathematical correctness. The lines are drawn from one contour curve to the other, all of the same thickness; and invariably separated from each other by one-fourth of the distance of the contours between which they are drawn. Thus, when the contour lines are very near each other, as in a steep rising ground, the shading lines are also close, and darken the shade at that part. When the

contours are distant from each other, so are the shading lines, and a faint shade is produced expressing a gentle slope. This method has been adopted in the maps drawn for the governments of France and the United States.

German maps.

3. The Saxon method. In this no contour lines are used. The slopes are all expressed by shading lines alone. The direction of these lines is always that of the greatest slope. A horizontal level has no shading. A piece of ground with a slope of 45° is all shade, that is, all black. For every difference of 5° in slope, a change is made in the proportion of light and shade, or white and black lines in a given space. Where the slope is 40°, the black is to the white as 8 to 1. At 35° slope, the black lines are to the white as 7 to 2; at 30° slope, as 6 to 3; at 25°, as 5 to 4, &c.; and at the horizontal level the map is white. Thus, where steep slopes rise, the shades are very dark, becoming rapidly lighter as the slopes diminish. The actual inclination is truly expressed, while the picturesque effect is preserved. This method is very generally used throughout Germany.

CHAPTER XIV.

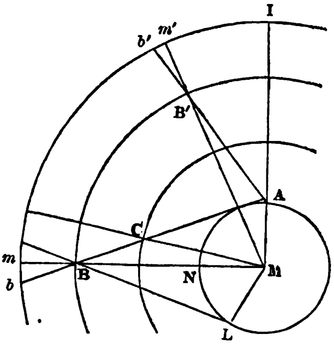
THE HEAVENLY BODIES.

(253.) The following paragraphs are intended to give some idea of the manner in which the distances, magnitudes, masses, densities, times of revolution, rotation, &c., of the distant heavenly bodies are deduced.

Parallax explained.

(254.) Before attempting to come to any conclusion as to the distance of any star, we must first know its *parallax*; that is, the difference of its apparent position when seen from a certain station on the *surface* of the earth and when seen from the *centre*; or, which is the same thing, the difference when seen from a position on the surface where it is not in the zenith, and one where it is so. Parallax, thus considered, is also the angle subtending the semidiameter of the earth at the distance of the celestial body. Suppose the sun to be in the horizon of a place A (fig. 67). It then appears to the observer at A, on the point *b* of the celestial sphere; while an observer, supposed to be at M, in the centre of the earth, or at N, directly under the sun, sees it at *m*. The arc *bm*, or the angle *m*B*b*=ABM, is the horizontal parallax, as it is called. When the sun has moved to B′, above the horizon, *b′m′* will be its parallax of altitude. In the zenith, the lines of vision from surface and centre coincide, and the parallax then becomes *nil*. Thus, parallax greatest at the horizon, diminishes in approaching the zenith; then it vanishes. It is always greater, too, the nearer the body observed

Fig. 67.



is to the earth, as may be seen from the figure, by comparing the parallax of two stars, one at B, the other at C. Parallax lowers the place of the stars — thus acting in a contrary direction to refraction. We see them less above the horizon from the surface than we should from the centre of the earth; and the greater is the apparent depression the nearer they are to the horizon. We can see only the stars in the zenith in the place in which they would be seen from the centre. The relations of the horizontal and altitude parallax of any star are easily deduced. Let the earth's radius be r, the distance of the star d, its zenith distance z, the horizontal parallax h, and the parallax of altitude H.

From the triangle BAM (fig. 67) we have

$$d : r = 1 : \sin h$$

and from the triangle B′AM we have

$$d : r = \sin z : \sin H$$
$$\therefore \sin z : \sin H = 1 : \sin h$$
$$\therefore \sin H = \sin h \times \sin z;$$

that is, the sine of the parallax of altitude = the sine of the horizontal parallax × sine of the zenith distance.

This formulæ renders it evident that parallax=o at the zenith, since the factor z=o there.

Horizontal parallax.

(255.) The horizontal parallax of the sun may be found by two observers stationing themselves, on the same day, at different places on the same meridian, A and L, and observing the noonday altitude B of the sun's centre. They thus obtain the sun's zenith distance at each station. As the fixed stars seen from *any* part of the earth have no parallax, if the sun were as distant as they, the two zenith distances, BMA, BML thus found, would be the sum of the latitudes north and south of the two observers: for they would together be equal to the angle AML, that is, the distance of the two stations across the equator. But since the sun's parallax increases these zenith distances, their observed sum will be equal to the latitude, plus the sum of the two parallaxes, that is, plus the angle ABL. By subtracting the sum of the latitudes from that of the zenith distances, we get ABL, the sum of the parallaxes; and by dividing ABL by the sum of the sines of the two zenith distances, we get the horizontal parallax. For if H and H^1 be the two parallaxes, h the horizontal parallax, z and z' the zenith distances, then (254)

$$\sin H = \sin h \times \sin z, \text{ and}$$
$$\sin H^1 = \sin h \times \sin z'$$

$$\therefore \sin \text{H} + \sin \text{H}^1 = \sin h \times (\sin z + \sin z')$$

$$\therefore \sin h = \frac{\sin \text{H} + \sin \text{H}.^1}{\sin z + \sin z'}$$

And since small arcs are proportional to these sines, we may say—

$$h = \frac{\text{H} + \text{H}.^1}{\sin z + \sin z'}$$

The following are the angles subtended by the earth's radius, at the earth's nearest distance to the bodies named:—

		Seconds			Seconds
Angle at the Sun .	=	8·58	Angle at Saturn . .	=	1
" Mercury	=	14	" Uranus . . .	=	·5
" Venus	=	31	" the Asteroids	=	4·5
" Mars .	=	21	" the Moon .	=	1°1′
" Jupiter	=	2			

Annual parallax.

(256.) The parallax by which observations are referred to the centre of the earth, is called the diurnal or geocentric parallax, to distinguish it from the annual parallax. This expresses the angle under which the diameter of the earth's orbit is seen from the centre of any star, or the apparent change of place in that star corresponding to an actual change of place made by the earth in its orbit. The diurnal parallax of Uranus, as we have seen, is half-a-second—his annual parallax is 30°. That of the other planets is greater the nearer they are. Annual parallax has been as yet found in the case of only three fixed stars. In the double star 61 of the Swan, it is about 0·3136″, which gives a distance of more than 52 billions of miles. Similarly, for two stars in the constellation Centaur, a distance of about 15 billions of miles has been calculated; and for the brightest star in the Lyre, a distance of above 65 billions.

Distance of the fixed stars.

(257.) The following consideration will prove that the fixed stars must be at some such distances from the earth. Let an observer walk round the circumference of a circle, 30 or 40 yards in diameter; and having fixed upon two distant objects, or rather points in those objects, let him measure with a good instrument the variations in apparent distance which these two points undergo, as he moves on different parts of the circle. A thoroughly good instrument will show him changes in the relative places of these points, even if they be distant from him by 100,000 times the diameter of the circle round which he walks. With the three exceptions above stated, similar observations have shown no change of place, no parallax of the fixed stars, while the earth moves round a circle of 190 millions of miles in diameter. Hence it undoubtedly follows that the distance of the fixed stars from us cannot be so small as 100,000 of the earth's diameters.

Calculation of the distance of a planet.

(258.) The parallax being found, the distance of the sun, moon, and planets, may be obtained. Let h be the horizontal parallax, r the radius of the earth, and d the required distance; then, figure 67,

$$\frac{\text{AM}}{\text{BM}}=\sin \text{ABM} \text{ or } \frac{r}{d}=\sin h \therefore d=\frac{r}{\sin h};$$

that is, the distance of a planet is the radius of the earth, divided by the sine of the horizontal parallax.

The distance of the sun is

$$d=\frac{4000}{\sin 8{\cdot}58''}=95,000,000 \text{ miles},$$

and of the moon,

$$d=\frac{4000}{\sin 61'}=240,000 \text{ miles nearly}.$$

(259.) Knowing the distance of one planet or celestial body, that of another may be found without the earth's radius; for the two distances are universally as the sines of the horizontal parallax of the respective bodies.

$$d=\frac{r}{\sin h} \text{ and } d'=\frac{r}{\sin h'}:$$

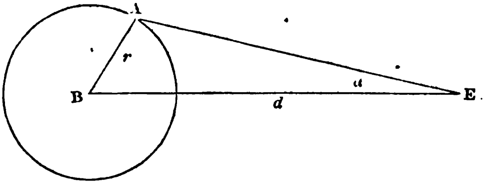
hence $\quad d : d' = \sin h' : \sin h.$

And since the angles are exceedingly small, the arcs may be taken for the sines; therefore,

$$d : d'=h' : h.$$

The parallax of the sun is about $\frac{1}{400}$th part of that of the moon, hence the sun must be 400 times more distant than the moon.

Fig. 68.



Calculation of the true diameter

(260.) The apparent diameter of a celestial body is the number of seconds which its disc subtends, as seen from the earth. This is measured by means of a micrometer. To find the true diameter from the apparent, let AB (fig. 68) be the true radius of a planet (r), AEB$=a$ be the apparent radius, EB$=d$ the actual distance, and EA a tangent to the planet at A; then

$$\frac{r}{d}=\sin a \therefore r=d \sin a;$$

that is, the true radius is equal to the actual distance, multiplied by the sine of the apparent radius. Hence the diameter is found. As the mean diameter of the sun is about 32′, and, therefore, radius 16′,

$$\sin 16' \times 95{,}000{,}000 = 441{,}000 \text{ miles nearly};$$

so that the radius of the sun is 112 times that of the earth.

The apparent mean radius of the moon is 15′ 33″.

$$\text{Sin } 15' \, 33'' \times 240{,}000 = 1{,}080 \text{ miles nearly.}$$

If the diameter of the earth be 1, that of the moon, therefore, will be ·273; or the diameter of the earth is 3⅔ times that of the moon.

and magnitude of a planet.

(261.) To determine the surface and solid contents of a heavenly body, from its distance and apparent diameter, we have the surface equal to four times the area of a great circle by spherical geometry. Therefore:

$$\text{Surface} = 4\, r^2 \times \pi = 4\, d^2 \times \pi \times \sin^2 a \text{ (260.)}$$

So, by spherical geometry, the solidity is

$$\frac{4\, r^3 \pi}{3} = \frac{4\, d^3 \pi \,.\, \sin^3 a}{3}.$$

(262.) The surface and solid contents of the heavenly bodies may easily be compared with those of the earth, since the surfaces of different spheres are to each other as the squares of their radii, and the solidity as the cubes.

The sun, therefore, has $112^2 = 12544$ times the surface of the earth; and $112^3 = 1404928$ times the solid content.

The earth, again, has 14 times the surface of the moon, and 49 times the solid content.

Rotation of the planets.

(263.) The time in which the sun or a planet rotates on its axis is determined by the time which a spot on its surface appears to depart from, and return to, the same position on the surface. A spot on the sun, for example, requires about 27 days to reappear to us on the very same part of the sun as before. But the earth in 27 days has advanced nearly 27° in its path in the same direction as that in which the sun rotates. Therefore the spot in 27 days has traversed not only 360° by the sun's rotation, but 27° more, before it could assume its previous position relative to the earth. In order to reduce for this excess, we have

387° a rotation plus excess : 360° a true rotation =
27 days, the time plus excess : x, the true time,

namely, 25 days.

Calculation of the length of their day,

(264.) On the surface of Mars, a spot is observed to reappear in the same place about 40 minutes later each day. Hence the period of one revolution of Mars on its axis, or one day of that planet, is found to be 24 hours and 37 minutes. For the earth in that day has moved 1°; thus we have 361° : 360° = 24 hrs. 40 m. : x = 24 hrs. 37 m.

and of their year.

(265.) The period of a planet's revolution in its orbit is found from the time between two successive similar positions of the planet in reference to the earth. In the case of the inferior planets, Venus and Mercury, it is found by the time between two successive equal departures on the same side from the sun; and of the upper planets, by the time between two successive oppositions. The third law of Kepler may also be applied for this problem. Mercury requires about 116 days for two successive equal departures from the sun — Venus about 584. But in 116 days the earth has advanced about 114°, and in 584 days, about 570°. Mercury, therefore, in 116 days has moved, not 360°, but 360° + 114° = 474°; and Venus, not 360°, but 360° + 576° = 936°. Hence the proportions

474° : 360° = 116 : x = 88 days.
936° : 360° = 584 : x = 224 days.

The year of Mercury, then, lasts 88, and that of Venus, 224 days.

The interval between two successive oppositions of Mars is about 2 years 52 days = 782 days; and its apparent course in that time is 409° 28′. The proportion
409° 28′ : 360° = 782 d. : x, gives 678 days for the value of x, and an approximate determination of the length of the year in Mars.

Mass of the planets.

(266). The determination of the mass or quantity of matter in the sun, or a planet, can only be inferred from their effects upon the motions of other celestial bodies. Thus the mass of the sun is estimated from its attraction on the planets; that of the earth, Jupiter, and Saturn, by their influence upon their own satellites; and that of Venus and Mars, by their disturbing influence upon the earth in its orbit. The mass of Mercury has been determined by Encke from the perturbations which the planet caused in the motions of the comet named after him.

Their density.

(267.) The density of a celestial body must evidently be found by dividing the mass of matter by the cubical space which it occupies. The mass of the sun is estimated at 354936 times that of the earth: its diameter, as we have seen, is 112 times that of the earth, and the magnitudes of the two bodies are as the cubes of their diameters. Therefore $\frac{354936}{112^3}$ gives ·26 for the density of the sun, that of the earth being 1.

Weight at their surfaces.

(268.) Weight, the effect of attraction, is, according to Newton's law, directly as the mass, and inversely as the square, of the distance. Since the mass of the sun is 354936 times that of the earth — the radius of the earth about 4,000 miles — and that of the sun about 112 times greater, or rather more than 44,000 miles, then taking weight on the earth's surface as unity, we have weight on the sun : weight on the earth =

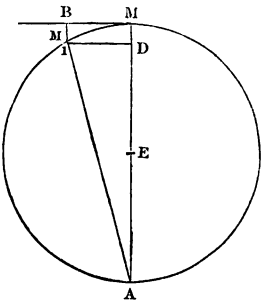
$$\frac{354936}{(44000)^2} : \frac{1}{(4000)^2} = 27{\cdot}9 : 1 :$$

hence it follows that the velocity of falling bodies is nearly 28 times greater on the sun's surface than on the earth's, or about 425 feet in a second; and that a pound of matter on the sun would press as heavily as 28 lbs. do on the earth. Similarly it is found that a pound of matter, at the earth's equator, would at the equator of Jupiter weigh 2·509 lbs., supposing Jupiter not to rotate. Taking the centrifugal force into account, its weight would be $\frac{1}{9}$th less.

Proof that gravitation extends to the planetary spheres.

(269.) That the same force which draws a stone to the earth extends to the celestial bodies, is evident from a consideration of the motions of the moon. Let MM_1A (fig. 69) represent the moon's orbit, E the earth, and MM_1 the part of the orbit described by the moon in one second. Draw the diameter MA, and MB a tangent to the orbit at M; also M_1B parallel to MA, and M_1D parallel to MB. M_1M being small, may be taken as a straight line, and is evidently the resultant of the propelling force MB, and the centripetal MD (136). MD, therefore, is the space through which the moon would fall towards the earth in the first second of time. Now $(M_1M)^2 = AM \times MD$ (Euc. vi.); and as AM and MM_1 are known, we may easily find MD.

Fig. 69.



For the whole circle is described by the moon in 27 days 43 m., or 2,360,580 seconds; and its circumference is equal to 60 times the circumference of the earth, or 7,926,336,000 feet. From these numbers we find the arc $MM_1=3,358$ feet. The diameter of the moon's orbit AM is found, from the same numbers, to be 2,523,031,140 feet. Hence $MD=\frac{(MM_1)^2}{AM}=\cdot00447$ feet. That is, a body at the distance of the moon would fall from a state of rest through this space in a second, if *acted upon by the force which keeps the moon in her orbit.* Let us now see through what space a body would fall *by gravity* at that distance. We know that at the distance of 1 radius it falls 16·09 feet in a second, and that gravity diminishes as the square of the distance increases. Then at the distance of 60 radii, we have for the space required $60^2 : 1^2 = 16\cdot09 : x = \cdot00447$ feet; exactly the same space through which it falls by the force that maintains the moon in her orbit.

(270.) Thus some power evidently impels the moon towards the earth; for if not, she would fly off along the tangent to her orbit. This power acts exactly as gravity at the distance of the moon would act. If it be not gravity, then the action of gravity must cease at some point between us and the moon, and a new power, *equal and similar* in its effect, come into operation. But why call by any other name that which does not differ from gravity?

DETAILS OF THE SOLAR SYSTEM.

	Mercury ☿	Venus ♀	Earth ⊕	Mars ♂	Vesta ⚶	Juno ⚵	Ceres ⚳	Pallas ⚴	Jupiter ♃	Saturn ♄	Hersch. ♅	Neptune ♆	Sun ☉	Moon ☽
Mean distances from the sun, the earth's being 10	4	7	10	15	24	27	28	28	52	95	192	309		
Periodic times	days 87	days 224	days 365	days 686	days 1335	days 1582	days 1681	days 1681	yrs. d. 11 315	years 29½	years 83	164½		
Times of rotation on axis	24h 5′	23h 30′	24h 0′	24h 31′					9h 56′	10h 16′			d. hrs. 25 10	days 27⅓
Inclination of orbit to ecliptic	7°	3° 24′		1° 51′	7° 8′	13° 5′	10° 37′	34° 37′	1° 46′	2° 30′	0° 46′	1° 47′		5° 8′
Inclination of planet's equator to its orbit		75°	23° 28′	29° 30′					3° 5′	28°			7° 10′	
Diameter, that of the earth being 1	0·39	0·97	1	0·52					11·56	9·61	4·26	4·8	112	0·27
Volume, that of the earth being 1	0·06	0·93	1·00	0·14					1470	887	77	110·6	1328460	0·02
Mass, that of the earth being 1	0·16	0·94	1	0·13					338	120	17	24	354936	0·013
Density, that of the earth being 1	2·77	1·04	1	0·93					0·25	0·13	·022	0·22	0·26	0·76
Diameter of the sun seen from the planets	80′	46′	32′	21′	13′	12′	11′	11′	6′	3′	1′	2″		
Velocity in miles per second	30	23	19	15	13	12	11	11	8	6	4	3		⅝

A TABLE SHOWING THE DAYS OF THE MEAN YEAR ON WHICH A WATCH KEEPING GOOD TIME OUGHT TO BE AN EVEN NUMBER OF MINUTES FASTER OR SLOWER THAN THE SUN'S NOON.

Days	Fast Minutes	Slow Minutes	Days	Fast Minutes	Slow Minutes
January . . 2	4	...	August . . 11	5	...
„ 4	5	...	„ 16	4	...
„ 6	6	...	„ 21	3	...
„ 8	7	...	„ 25	2	...
„ 11	8	...	„ 29	1	...
„ 13	9	...	September . 1	0	...
„ 16	10	...	„ 4	...	1
„ 19	11	...	„ 7	...	2
„ 22	12	...	„ 10	...	3
„ 27	13	...	„ 13	...	4
February . 1	14	...	„ 16	...	5
„ 21	14	...	„ 19	...	6
„ 28	13	...	„ 22	...	7
March . . 5	12	...	„ 24	...	8
„ 9	11	...	„ 27	...	9
„ 12	10	...	„ 30	...	10
„ 16	9	...	October . . 4	...	11
„ 19	8	...	„ 7	...	12
„ 23	7	...	„ 11	...	13
„ 26	6	...	„ 15	...	14
„ 29	5	...	„ 20	...	15
April . . . 1	4	...	„ 28	...	16
„ 5	3	...	November . . 16	...	15
„ 8	2	...	„ 21	...	14
„ 12	1	...	„ 25	...	13
„ 16	0	...	„ 28	...	12
„ 20	...	1	December . . 1	...	11
„ 25	...	2	„ 3	...	10
May . . . 1	...	3	„ 6	...	9
„ 15	...	4	„ 8	...	8
„ 30	...	3	„ 10	...	7
June . . . 5	...	2	„ 12	...	6
„ 11	...	1	„ 14	...	5
„ 16	...	0	„ 17	...	4
„ 20	1	...	„ 19	...	3
„ 25	2	...	„ 21	...	2
„ 30	3	...	„ 23	...	1
July . . . 5	4	...	„ 25	...	0
„ 11	5	...	„ 27	1	...
„ 22	6	...	„ 29	2	...
			„ 30	3	...

ETYMOLOGY OF TERMS.

Aphelion	ἀπὸ, at a distance from; ἥλιος, the sun.
Apogee	ἀπὸ, and γῆ, the earth.
Azimuth	from an Arabic word of similar sound and meaning.
Calendar	*calendæ,* the first day of each month among the ancient Romans.
Cardinal	*cardo,* a hinge. Hence *cardinalis,* principal.
Colures	*coluri,* the Latin name for *all* great circles passing through the poles.
Cycle	κύκλος, a circle.
Eclipse	ἔκλειψις, a failing (of light).
Ecliptic	i. e. the line in which eclipses occur.
Epact	ἐπακτὸς, added; so that the epact is the *excess* of the solar year, above the lunar year, of twelve lunations.
Equinox Equinoctial	*æquus,* equal; *nox,* night. Equinox refers to the *time* when the sun is on the equator; equinoctial to the *place* or point of the equator in which the ecliptic cuts it.
Horizon	ὁρίζων, a boundary.
Nutation	*nutatio,* a nodding.
Parallax	παράλλαξις, change produced by motion from one point to another.
Perigee	περί, about, near; γῆ, the earth.
Perihelion	περὶ and ἥλιος.
Planet	πλανήτης, a wanderer.
Solstice	*solstitium,* from *sol,* the sun, and *stare,* to stand.
Syzygies	συζυγία, a yoking together. At the syzygies the sun, earth, and moon are in one straight line.
Stereographic	στερεὸς, solid; γράφειν, to draw.
Tropic	τρέπειν, to turn.

EXERCISES.

1. Explain fully the terms Zenith, Nadir, Horizon, Meridian, Celestial Equator.

2. From what fixed lines are geographical Latitude and Longitude measured? From what fixed lines, celestial Latitude and Longitude?

3. On what fixed lines are Declination and Right Ascension measured?

4. Enumerate the Signs of the Zodiac from Aries, and write down the marks by which they are represented.

5. Distinguish between the Signs and Constellations of the Zodiac.

6. Enumerate the different methods of determining the Longitude.

7. Explain fully the terms Equator, Equinoctial, Equinox, Altitude, Solstice, Tropic, Azimuth, Colure.

8. Describe the apparent daily revolution of a star with the polar distance of 45°, as seen from London.

9. Investigate a method of finding the Latitude from the Altitudes of a celestial object near the meridian. Show how the Hour-angles* are computed from the 'times by chronometer' for the sun or a fixed star.

---

* The hour-angle is the angle formed at the pole, and measured on the equinoctial, between the meridian circle of a place, and the declination circle of a star or the sun (49). When, by the daily revolution of the celestial sphere, the declination circle coincides with the meridian, that angle is 0, and the star or the sun culminates: e. g., What is the hour angle when a star is yet 1 hour 40 minutes from its culmination? According to (65),

$$1 \text{ hour} = 15^\circ, \text{ and } 40' = 10^\circ; \text{ therefore}$$

the hour-angle of the star at the time of observation is $15^\circ + 10^\circ = 25^\circ$. Again, if the hour-angle, after culmination, be 32° 20′, what time has elapsed since the culmination?

$$15^\circ : 1 \text{ hour} = 32^\circ\ 20' : 2 \text{ h. } 9 \text{ m. } 20 \text{ s.}$$

10. Prove the equality of the Latitude to the Altitude of the Pole.

11. What must be the Polar distance of a star which at its lower culmination just touches the horizon of London?

12. Give one reason, and that the simplest, 1st, for believing the earth not to have an infinitely extended surface; 2ndly, for believing it to be a sphere.

13. Explain why twilight is shorter in the Tropical regions than in latitudes farther north or south.

14. Explain the seasons, illustrating your remarks by a figure.

15. Explain the phases of the moon, similarly.

16. Distinguish between a sidereal and a synodical month.

17. What are the seasons of the Tropical regions?

18. Distinguish (*a*) the sidereal day; (*b*) the solar day; (*c*) the mean solar day; (*d*) the lunar day.

19. What are the two causes of the difference of true and mean Solar Time?

20. What would be the prevalent currents of the atmosphere and ocean if the earth were at rest? What change would be produced in them by the diurnal motion if there were no annual motion? What effects would result from the annual, in addition to the diurnal motion?

21. If there were no land on the earth's surface, the currents of the air and the ocean would always have an easterly direction near the Equator, and a westerly direction towards the Poles. Why is this?

22. Assign reasons for believing the earth to revolve on its axis; and for believing it to have an annual motion.

23. State the arguments for the Spheroidal form of the Earth.

24. Explain the confusion of dates to which circumnavigators, as they sail eastward or westward round the world, are liable.

25. Define the rational and sensible horizons. Why are they the same at the distance of the fixed stars? Prove that the dip of the visible horizon varies with the square root of the height of the eye.

26. By what astronomical causes is the extreme summer heat of different latitudes influenced? Show that these have a tendency to counteract each other.

27. Explain the changes in the length of the day as we travel from the equator towards the North Pole, at the winter solstice.

28. Why is the number of sidereal days in a tropical year one more than the number of solar days?

29. State distinctly the object of a perfect Calendar. Explain

the provisions of the Gregorian, and give a sketch of the history of its adoption by different nations.*

30. What is the altitude of the sun at London on December 22, June 22, March 21, and September 23?

31. Prove that the like parts of two parallels of latitude have the same ratio as the cosines of their latitudes.

32. If, in latitude 45°, I travel 1,000 miles due east, by how many degrees shall I have changed my longitude?

33. Explain the means by which the Longitude of a place is determined by the Chronometer.

34. How is it known that the fixed stars are greatly more distant from us than the sun and planets?

35. Describe the apparent diurnal motion of the stars, to an observer at the Equator, and to one at the Poles.

36. Explain the different lengths of day and night in England, at different seasons of the year.

37. What is the difference between the sidereal *time* and apparent solar time? When do they agree?

38. Explain generally what is meant by the 'Equation of Time,' its causes, and how to make use of it from the Almanack.

39. What are the methods for determining the shape of the earth, and what are the causes of the discrepancies observed in the results of these methods?

40. Describe the method of measuring a part of a meridian line on the surface of the earth.

41. Explain the causes of the difference in the weight of the same body at the Equator and at the Poles.

42. Describe Foucault's Pendulum Experiment.

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* In the year 1793 a new calendar was adopted by the government of the French Republic. The year was divided into twelve months of thirty days each. Five days were added at the end, which, under the absurd name of *sansculottides*, were celebrated as festivals. Every fourth year a sixth complimentary day was added. Each period of four years was called a Franciade. The years 100, 200, 300, of the Republic were to be common years; 400 to be a leap, or sextile year. Every four centuries were to repeat themselves in a similar manner, till the fortieth, which was to terminate with a common year. The months were divided, not into weeks, but into three parts, called decades, of ten days each. Sept. 22 of the Gregorian calendar was the beginning of the new month *Vendémiaire*; Oct. 22, of *Brumaire*; Nov. 21, of *Frimaire*; Dec. 21, of *Nivose*; Jan. 20, of *Pluviose*; Feb. 19, of *Ventose*; March 21, of *Germinal*; April 20, of *Floréal*; May 20, of *Prairial*; June 19, of *Messidor*; and Aug. 18, of *Fructidor*. The use of this calendar ceased with the year 1805.

43. With what velocity do the parts of the earth along the 45th degree of latitude rotate?

44. In what respect have parallax and aberration opposite effects?

45. Show by a figure the different effects of aberration in different parts of the earth's orbit.

46. The surface of the globe may be divided into hemispheres, of which one contains nearly all the land, and the other the greater portion of the water. Illustrate this division by a diagram.*

47. In what direction is the great normal current of the atmosphere?

48. What is meant by the Trade Winds? Explain the permanency of their direction.

49. Explain why the interval between the vernal and the autumnal equinox is about seven days longer than between the autumnal and vernal.

50. Show how to draw a meridian line by the shadow of a vertical gnomon on a horizontal plane.

51. Explain the causes of the apparent retrograde motions of the planets.

52. Explain the phases of Venus, the changes in the apparent diameter of that planet, and the reason of its being sometimes the evening, sometimes the morning star.

53. When Venus is at its greatest elongation, what is the relation of the line along which it is seen from the earth to the orbit of the planet?

54. State the three Laws of Kepler.

55. State Bode's Law of Distances.

56. Name the planets in our system; give some account of their distances from the sun. Which of them have satellites? To what practical uses are observations upon these satellites applied?

57. Give an account of the phenomena of Refraction.

58. Describe the method of ascertaining the longitude by the meridian passage of the moon.

59. Explain in a simple manner the principle of the method of finding the longitude by lunar observations. How may a chronometer be regulated by Lunars?

60. When will an error in the observed altitude produce the least effect on the time deduced from it, and why?

---

* A map of the world in hemispheres, projected on the horizon of London, would serve for such a diagram, and show that London stands nearly in the centre of the terrestrial hemisphere.

61. Describe the Mariner's Compass. What is meant by its variation?

62. Prove that the radius-vector of a planet's orbit sweeps over equal areas in equal times.

63. State the Law of Gravitation. Through what number of feet will a body near the surface of the earth fall in one, two, three, &c., seconds?

64. Explain the causes of Eclipses of the Sun and Moon; and of the different kinds of eclipses. Why do they not occur every month?

65. Why does the moon present always the same face to us?

66. How would you show from observations of the moon, day by day, that this luminary moves round the earth in about twenty-eight days? If the moon rises at 10 P.M., is she on the wane, or increase?

67. Explain why the moon is, in our latitude, occasionally nearer to the zenith than the sun can ever be.

68. What points on the earth's surface have 14 days' moonlight and darkness alternately?

69. At what rate per hour is a spot on the equator of the moon carried round by its rotation on its axis; the diameter of the moon being 2,160 miles?

70. Explain the Harvest Moon.

71. Explain the theory of the Tides, and give the causes of the diurnal and two semi-diurnal, as well as the spring and neap tides.

72. Why does the time of high water at any place vary daily? After what interval is it again the same at that place, and why?

73. Describe the progress of the Tide-Wave, and the cause of the Tidal Bore.

74. Describe, and state the cause of, the nutation of the Earth's Axis.

75. What are the Metonic, Solar, and Chaldaic cycles?

76. What was the Dominical Letter of the year 763, B.C.? E, that is, the fifth day of January was Sunday.

77. What will be the Dominical Letter of the year 1901? F. Of the year 1896? E and D; 1896 being a leap year. (158.)

78. On what day of the week will July 18, 1873, fall? *

---

* Find the Dominical Letter for 1873, which is E.; E being Sunday, the day for which A stands, or the 1st of January, will be Wednesday. Write down all

79. What is the Epact? Show its use in ascertaining the age of the moon on a given day.

80. What is meant by Precession of the Equinoxes? Describe its effect on the position of the pole-star.

81. The Earth being assumed to be an oblate spheroid, what is the accurate direction of Gravity?

82. State the defects of the Orthographic and Stereographic projections of the Sphere.

83. Investigate a method of finding the distance between two places on a great circle.

84. What are the advantages of great circle sailing? And what are the practical objections to it?

85. Explain the principle upon which Mercator's Chart is constructed, and the uses of the chart.

86. What are Meridional parts? What is the value of 1 second of latitude on Mercator's Chart, in latitude 60°?

87. Investigate the formula for computing the Meridional parts.

88. Investigate the formula for finding the Horizontal Parallax of the sun, by observations made at two places situated on the same meridian.

89. How may it be proved that the Law of Gravitation extends at least to the circumference of the Moon's Orbit?

90. What is the maximum amount of Refraction? Where is a star situated when under its greatest influence? Where, when its rays suffer no refraction?

91. How is refraction connected with twilight?

the days of the year which can have A for their letter. These are shown in the following Table:—

January	1,	8,	15,	22,	29
February	5,	12,	19,	26,	—
March	5,	12,	19,	26,	—
April	2,	9,	16,	23,	30
May	7,	14,	21,	28,	—
June	4,	11,	18,	25,	—
July	2,	9,	16,	23,	30
August	6,	13,	20,	27,	—
September	3,	10,	17,	24,	—
October	1,	8,	15,	22,	29
November	5,	12,	19,	26,	—
December	3,	10,	17,	24,	31

We have thus the date of all the Wednesdays in the year 1873. July 16 is one of these, therefore July 18 will fall on Friday.

92. Explain the cause of the apparently elliptical form of the moon when it is near the horizon.

93. Explain generally why the magnitude of a Solar Eclipse is, and of a Lunar is not, affected by the geographical position of the observer to whom the eclipse is visible.

94. Trace the steps by which we have arrived at a knowledge of the form, magnitude, and motions of the Earth.

95. Show that the Equation of Time vanishes four times a year.

96. Describe a method of finding the distance of the Sun from the Earth.

97. Describe the apparent path of the Sun in the heavens with respect to the fixed stars, and to the Equinoctial. Is the Sun always on the meridian of a place when it is 12 o'clock at that place?

98. When is a planet's motion said to be retrograde? Show through what portion of its orbit an inferior planet's motion will appear (1) progressive, (2) retrograde, (3) stationary; the orbits of the earth and planet being supposed circular, and in the same plane.

99. Show how to calculate the value of the centrifugal force due to the earth's rotation in any latitude. Is the observed change in gravity due entirely to the centrifugal force thus determined?

100. Assuming that the moon is retained in her orbit by the force of gravity, show how to perform the necessary calculations for determining the numerical measure of gravity at the earth's surface, the moon being supposed to move in a circle round the earth at rest.

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